Traffic-Aware Resource Allocation and Feedback Design in Wireless Networks)

Apostolos Destounis Destounis

N° d'ordre : 2014-13-TH

THÈSE DE DOCTORAT

DOMAINE: STIC

SPÉCIALITÉ: Télécommunications

École doctorale "Sciences et Technologies de l'Information, des Télécommunications et des Systèmes"

Présentée par :

Apostolos Destounis

Sujet:

Allocation de ressources et conception du feedback dans les réseaux sans fil avec prise en compte du trafic

(Traffic-Aware Resource Allocation and Feedback Design in Wireless Networks)

Soutenue le 9 Mai 2014 devant les membres du jury:

M. Michel Kieffer,	CNRS/Supélec	Examinateur, Président du Jury
M. Vincent K. N. Lau,	Hong Kong University of Science and Technology	Rapporteur
M. Alexandre Proutière,	Royal Institute of Technology	Rapporteur
M. Anthony Ephremides,	University of Maryland	Examinateur
M. Leandros Tassiulas,	CERTH/University of Thessaly	Examinateur
M. Mohamad Assaad,	Supélec	Encadrant de Thèse
M. Bessem Sayadi,	Alcatel-Lucent Bell Labs France	Encadrant de Thèse
M. Mérouane Debbah,	Supélec	Directeur de Thèse

Abstract

Wireless systems are facing an increase in the data demands, and this trend is expected to continue in the future. This increase is mostly due to demand of video and data services. The most prominent approaches proposed to deal with this problem, namely the use of multiple antennas and OFDMA modulations (already part of the 3GPP LTE standards) and Small Cell Networks have mostly been analyzed from a pure physical layer perspective, focusing on metrics like total system throughput. However, the traffic pattern of video and data requests as well as the individual requests of the users have to be also taken into account when designing resource allocation algorithms. The objective of this thesis is, therefore, to study the impact of physical layer resource algorithms (power control, precoding, scheduling) and CSI feedback on the behaviour of the queues of the users. In particular, we study the problems of precoding and power control to regulate the behaviour of the users' queues in the interference channel, as well as joint feedback/training and user selection and scheduling in order to stabilize the queues for a large area of traffic demands in the MISO and OFDMA broadcast channels. To this end, we use tools from heavy traffic asymptotic modelling of communication networks and stochastic stability theory.

Abstract (French)

Les réseaux sans fil sont confrontés à une augmentation croissante en demande de données, qui devrait continuer à croitre dans les années à venir. La raison principale de cette croissance est liée à la demande en services vidéo et données. Les plus importantes approches proposées pour faire face à ce problème, notamment l'utilisation des antennes multiples, le codage OFDMA (qui font déjà partie des standards 3GPP et LTE), et le déploiement de réseaux à petites cellules, ont été examinées plutôt d'un point de vue couche physique, en se concentrant sur des mesures de performance tel que le débit total du système. Cependant, les caractéristiques du trafic vidéo et des données ainsi que les demandes individuelles des utilisateurs doivent être prises en compte pour la conception des algorithmes d'allocation de ressources radio. L'objectif de cette thèse est d'étudier l'impact des algorithmes d'allocation de ressources radio (contrôle de puissance, pré-codage, ordonnancement) ainsi que les informations concernant l'état du canal sur le comportement des files d'attente des utilisateurs. Nous étudions, en particulier, le problème de pré-codage et de contrôle de puissance dans le canal d'interférence, dans le but de réguler le comportement des files d'attente des utilisateurs et conjointement la rétroaction/estimation de canal et la sélection et ordonnancement des utilisateurs. Ceci afin d'assurer la stabilité des files d'attentes pour une grande partie des demandes de trafic dans les systèmes de diffusion MISO-OFDMA. Pour assurer cela, nous utilisons des outils mathématiques de la théorie des modèles asymptotiques "heavy traffic" et de la théorie de la stabilité stochastique.

Résumé en français

Introduction et Problématique

Récemment, les réseaux cellulaires sans fil ont connu une demande croissante en données par les utilisateurs et cela une devrait continuer durant les prochaines années. Cette augmentation est due à la prolifération des smartphones et tablettes qui ont induit une demande élevée en applications données et vidéos.

- **Réseaux à petites cellules :** Les réseaux à petites cellules sont des réseaux denses, comportant des petites stations de bases de couverture limitée, de faible puissance de transmission et réutilisation de fréquence [1]. Le but de ces réseaux est d'assurer une meilleure couverture et réutilisation des fréquences tout en augmentant la capacité du réseau. Cependant, le déploiement dense de petite station de bases fonctionnant sur la même fréquence nécessite des techniques de gestion d'interférences. Dans la littérature, différents travaux ont prouvé que la coopération entre les stations de base permet de réduire l'interférence intercellulaire et d'augmenter l'efficacité spectrale [2], [3]. Hors, en pratique ce n'est pas toujours possible d'atteindre ces résultats théoriques en raison des limitations de la capacité des liaisons entre les stations de base. De plus, dans les cas réels, les utilisateurs soumettent des requêtes, qui arrivent du réseau de base avec des motifs aléatoires et des exigences élevées de la part des utilisateurs en termes de qualité de service (QdS) tels que : le délai, perte de données. . . etc. Une meilleure approche serait donc d'allouer les ressources radio aux stations de base en s'adaptant aux caractéristiques du trafic en temps réel ainsi que qu'à l'état des files d'attente. A titre d'exemple, une station de base dont les utilisateurs ont peu ou pas de données en attente d'envoi peut émettre avec une puissance très faible pendant une certaine durée et ainsi éviter les problèmes de congestion au niveau des stations de bases voisines qui eux ont des demandes plus urgentes de la part des utilisateurs. Cela semble une approche prometteuse au lieu de se concentrer (seulement) sur les paramètres traditionnels de la couverture et de l'efficacité spectrale.

- **Utilisation de plusieurs antennes au niveau des stations de base :** Les avantages de l'utilisation de plusieurs antennes dans les systèmes sans fil mono- et multi- utilisateurs sont maintenant très bien compris [4, 5], principalement d'un point de vue efficacité spectrale. Dans les réseaux à petites cellules, les stations de base peuvent utiliser plusieurs antennes pour mieux gérer les interférences [3]. Dans les systèmes avec une cellule unique, l'utilisation d'antennes multiples permet à la station de base de servir plusieurs utilisateurs dans le même bloc de temps-fréquence, améliorant ainsi la performance du système. Cependant, afin de bénéficier pleinement du potentiel du MIMO multi-utilisateur, une connaissance précise de l'état des canaux des utilisateurs est nécessaire. Le moyen le plus important de le faire est considéré comme l'estimation, dans le mode duplex par séparation temporelle (TDD), par les utilisateurs, c'est-a-dire que les utilisateurs envoient des séquences orthogonales pour que la station de base estime leurs canaux. Cette technique exploite la réciprocité du canal et a l'avantage que la longueur des séquences d'entraînement est indépendante du nombre d'antennes de la station de base. Cette observation a conduit au concept prometteur de " Massive MIMO", où les stations de base sont équipées avec beaucoup plus d'antennes que les utilisateurs [6, 7]. Cependant, la longueur des séquences de pilotes doit s'adapter proportionnellement au nombre d'utilisateurs actifs. Étant donné que le temps de cohérence (ou la longueur de l'intervalle de temps utilisé par le protocole) est limité, ayant de nombreux utilisateurs actifs dans un intervalle signifie que le temps consacré à la transmission des données est faible. Par conséquence, la sélection des utilisateurs doit être effectuée de manière dynamique, en prenant en compte cette marge, les caractéristiques du trafic et l'état actuel des files d'attente de chaque utilisateur.

- **Ordonnancement des utilisateurs et utilisation des canaux parallèles :** Exploiter les variations temporelles des canaux avec évanouissement (un concept appelé "la diversité multi-utilisateurs") pour augmenter l'efficacité spectrale en sélectionnant les utilisateurs ayant de bonnes conditions canal [8] est également très bien compris dans les systèmes multi-utilisateurs. Cette idée, dénommée "ordonnancement opportuniste", peut fournir une haute efficacité spectrale et aussi assurer l'équité entre les utilisateurs [9], et a été déjà utilisée dans les systèmes 3G [10]. Créer des canaux parallèles en fréquence, en utilisant des modulations OFDMA, ou en espace, par l'utilisation de plusieurs antennes à formation de faisceau orthonormé, peut améliorer ce concept. En fait, les deux (OFDMA et MIMO avec la formation de faisceau orthonormé) font partie des dernières normes LTE et LTE-A pour les communications cellulaires [11]. Contrairement aux deux autres cas (c'est-à-dire l'utilisation de plusieurs antennes et la réduction de la taille des cellules), l'ordonnancement prenant en compte le trafic et les files d'attente des utilisateurs a été largement étudié [12] et reste un sujet de recherche très actif, spécialement après le travail séminal [13] dont les auteurs ont introduit l'ordonnancement de type "MaxWeight". Toutefois, la moitié des travaux supposent que la station de base a une connaissance parfaite des canaux des utilisateurs. Cela ne peut être acquis que par la rétroaction des récepteurs. Ce qui nécessite des ressources en temps (dans les systèmes TDD) ou en fréquence (dans les systèmes FDD), qui sont limités et pourraient être utilisés pour la transmission. La surcharge de rétroaction est encore plus grande dans le cas des systèmes utilisant des canaux parallèles, conduisant à un compromis exploration-exploitation entre l'utilisation des ressources pour obtenir des informations sur de nombreux utilisateurs (sélectionnant ainsi des utilisateurs avec de bons canaux pour transmettre) et ayant moins d'utilisateurs renvoyant l'état de leurs canaux, réduisant ainsi la surcharge et utilisant plus de ressources pour la transmission des données.

Motivés par les considérations ci-dessus, cette thèse se focalise sur les interactions entre la couche physique et la couche de contrôle d'accès (MAC). Plus précisément, nous étudions comment la répartition des ressources de la couche physique (contrôle de puissance, précodage, ordonnancement) et de rétroaction affectent les performances de la couche MAC du système. Celui-ci est modélisé par le comportement des files d'attente dans les stations de base, où les données des utilisateurs sont stockées en attente de transmission. Des algorithmes d'allocation de ressources radio pour améliorer cette performance sont proposés. Deux mesures de performance sont examinées : (i) la probabilité que la longueur de la file d'attente dépasse un certain seuil et (ii) la région de la stabilité du système, qui signifie en pratique l'ensemble des demandes des utilisateurs en termes de taux d'arrivée que le système peut supporter tandis que les utilisateurs connaissent des retards finies. La pertinence du premier objectif est de contrôler la perte de données due aux débordements de file d'attente et/ou au délai résultant du stockage des données dans la station de base. Le deuxième objectif est pertinent pour les utilisateurs qui demandent des services de données, car l'élargissement de la zone de stabilité implique, dans un sens, que les utilisateurs peuvent demander des téléchargements d'un volume plus élevé. D'un autre côté, les méthodes utilisées pour l'agrandissement de la région de stabilité du système peuvent être également utilisées d'une manière simple pour améliorer les performances du système lorsque celui-ci est mesuré en fonction du débit moyen par utilisateur [14, 15]. Ces méthodes ont été employées avec des résultats prometteurs dans des travaux récents [16, 17], qui se sont focalisés sur le streaming vidéo dans les réseaux sans fil.

Cette thèse aborde les problèmes de contrôle des ressources radio dans les systèmes sans fil, en considérant le fait que chaque utilisateur demande un processus dynamique de trafic. Les deux principaux aspects étudiés dans cette thèse sont : (i) le contrôle de puissance et précodage pour le canal d'interférence de telle sorte que la probabilité que la file d'attente de chaque utilisateur dépassant un seuil est à une valeur désirée et (ii) le feedback et sélection/ordonanncement des utilisateurs dans les systèmes de liaison descendante de cellules simples TDD (c'est à dire diffusé canaux avec la rétroaction faite en TDD) afin d'agrandir la zone de stabilité du système.

Contrôle de Puissance et Précodage dans le canal d' Interférence MISO

Dans ce chapitre, nous considérons un système avec K émetteurs, fonctionnant dans la même bande de fréquence, ayant chacun L antennes et servent un seul récepteur. Le défi ici est que les liens interfèrent les uns avec les autres de sorte que la longueur de la file d'attente d'un émetteur dépend aussi sur l' allocation de la puissance (et précodeurs) des autres émetteurs. Ce modèle peut correspondre à un réseau de petites cellules employant la même fréquence porteuse pour servir les utilisateurs. L'objectif initial est de minimiser la puissance totale dans un laps de temps infini de telle sorte que la probabilité que la taille de la file d'attente de chaque émetteur dépassant un seuil est fixé. Cet objectif peut correspondre à la fixation d'une probabilité de perte de données (pour les tampons finis aux émetteurs) dans certaines valeurs souhaitées ou une exigence de retard, ce qui est un aspect crucial pour les applications multimédias. En cas de retard-contraint le nombre (moyen) de retard est proportionnel à la longueur de file d'attente en raison de la loi de Little. Par conséquent nous pouvons avancer que la délimitation du retard inférieur à un seuil désiré est dans un sens équivalent à la délimitation de la longueur de la file d'attente en dessous d'un seuil correspondant à la limite du délai. De plus, le trafic entrant et les files évoluent de façon aléatoire cependant, une telle contrainte n'est pas possible a tenir. Nous allons donc considérer la métrique probabiliste suivante pour la longueur de la file d'attente sur chaque émetteur

$$\mathbb{P}\left\{q_k(t) > q_k^{thr}\right\} = \delta_k, \tag{1}$$

qui signifie fixer une probabilité d'interruption de la mémoire tampon dans certaines valeurs qui peuvent être tolérées par l'application (ce qui implique également que le retard sera assez petit la plupart du temps). Nous abordons le problème en proposant une stratégie de contrôle de puissance de sorte que ces contraintes soient respectées, en utilisant la modélisation asymptotique "heavy traffic". L'approche est alors de diviser la puissance en deux parties : l'équilibre et la réserve. Le problème de la puissance d'équilibre consiste à allouer de la puissance en fonction des états du canal de telle sorte que, en moyenne, la débit de transmission soit égal au débit d'arrivée. Dans la littérature, ce type de problèmes a été largement étudié pour les systèmes à unique et multiple antennes (on peut se référer à [18], [19] pour plus de détails). Le principal défi réside dans la modélisation et l'attribution de la réserve de puissance. Une approche immédiate pour contrôler de façon optimale la puissance de réserve serait de formuler le problème en utilisant la théorie de la commande optimale et des équations Hamilton-Jacobi-Bellman (HJB). Cependant, les contraintes (1) rendent le problème très difficile et les solutions ne sont pas assurées d'être faciles. En outre, le système (sous certaines politiques de contrôle) peut ne pas être ergodique. Même si le système est stationnaire et ergodique, trouver une expression analytique de la fonction de distribution stationnaire de l'évolution de la file d'attente n'est pas facile. Cela est dû à l'interaction entre les files d'attente des différents utilisateurs par l'intermédiaire du brouillage. Dans ce chapitre, nous abordons le problème ci-dessus comme suit. Nous montrons tout d'abord que les files d'attente des utilisateurs dans le régime "heavy traffic" peuvent être modélisées comme une équation différentielle stochastique multidimensionnelle avec des réflexions. Puis, on profite de la structure spécifique de la matrice de la réflexion et nous proposons une politique de contrôle qui découple l' équation multidimensionnelle en plusieurs équations différentielles stochastiques simples en parallèle et assure une mesure invariante pour chacune de ces équations. En utilisant des résultats de la théorie des probabilités, nous arrivons a obtenir une expression analytique de la fonction de distribution stationnaire de la dynamique de chaque équation, qui permet de trouver une relation entre la puissance de réserve affectée et la probabilité de débordement dans (1). Notez que la valeur de la réserve de puissance allouée par notre algorithme par rapport à la puissance d'équilibre est très faible. En d'autres termes, l'écart d'optimalité sous notre approche de réserve de puissance et d'autres approches de contrôle optimales (par exemple en utilisant les équations HJB) est faible dans de nombreux scénarios.

En ce qui concerne le problème des transmissions simultanées sur des canaux interférents, des travaux considérables ont été faits dans l'allocation d'énergie de sorte que le SINR à chaque récepteur soit supérieur à un seuil déterminé. Dans [20], un tel algorithme, qui est totalement distribué, a été proposé et dans les années suivantes, des modifications et extensions ont été apportées; pour une étude approfondie des algorithmes de commande de puissance avec cible SINR se référer à [18] et références qui y sont. Cette approche n'est cependant pas adaptée à la nature du trafic des données et les applications de streaming vidéo, car ils ne s'adaptent pas à la circulation, les états des files d'attente et/ou les demandes spécifiques de l'application. Dans ce contexte, dans [21], un ordonnanceur basé sur la théorie de commande H-infini a été proposé afin de réguler les tampons de petites stations de base de cellules autour d'une longueur de cible. Dans [22], le problème du contrôle de puissance pour le streaming vidéo au débit variable sur un réseau cellulaire est concerné, avec l'hypothèse que les vidéos demandées soient stockées dans les stations de base (donc la dynamique de la circulation stochastiques ne sont pas prises en compte). Les auteurs étudient le problème de maximisation du débit sous contraintes et débordement bas au playout des tampons des récepteurs et proposent une centralisation optimale et un algorithme décentralisé proche de l'optimal sous certaines hypothèses de faisabilité pour la SINR.

Les auteurs de [23] proposent de l' ordonnancement dynamique (à l'intérieur de la cellule) et l'allocation de puissance (pour gérer les interférences intercellulaires) ainsi que la minimisation du délai moyen dans le système en fonction de la longueur des files d'attente. Le problème est formulé comme un Processus de Décision de Markov, et un algorithme d'apprentissage en ligne est utilisé pour proposer la solution. L'algorithme proposé est semi-décentralisée dans le sens qu' un contrôleur central fixe les niveaux de puissance de transmission, mais la décision d'ordonnancement est prise au niveau de chaque Station de Base. Pour une étude de l'utilisation de ces outils dans les problèmes d'allocation de ressources voir [24] et les références qui y sont. Cependant, dans ces problèmes, l'objectif est d'optimiser une fonction unique soumis à des contraintes sur les esperences des files d'attente, qui sont plus faibles que les probabilités de dépassement que nous considérons ici. En outre, ces techniques nécessitent la résolution de l'équation de Bellman qui en général peut être résolu hors ligne que numériquement à un coût de calcul élevé, et des algorithmes d'apprentissage pour la mise en œuvre en ligne peuvent converger lentement.

Un autre axe de travail en ce qui concerne l'allocation des ressources dans les réseaux sans fil se fait en utilisant des techniques de dérivation de Lyapunov (voir [25], [24] et les références citées). Ces travaux portent sur le problème de la minimisation d'une fonction de coût pour le réseau tout en gardant les files d'attente stables. Toutefois, dans notre travail, nous sommes intéressés par la satisfaction individuelle des contraintes de Qualité de Service pour chaque utilisateur, sous une forme beaucoup plus puissante par rapport aux travaux où uniquement la stabilité des files d'attente est exigée.

L'approche suivie dans ce chapitre est basée sur la modélisation asymptotique du “”heavy trafic” d'un réseau. Initialement utilisée pour l'analyse des files d'attente et des réseaux de files d'attente, elle consiste à examiner le comportement du système dans le cas ou les taux des arrivées deviennent presque aussi grand que le débit de service. Il s'avère que les modèles dans ce cas asymptotique deviennent plus dociles et leur étude peut révéler des informations utiles pour le comportement du système, même si cette condition n'est pas satisfaite en pratique. En outre, en raison de sa docilité, le régime asymptotique du “heavy trafic” peut être utilisé pour trouver analytiquement une politique de contrôle dans le réseau (par exemple, la politique de routage, la planification de la transmission, le réglage du taux de service, ...); cette politique peut ensuite être appliquée dans les cas où le réseau n'est pas nécessairement très chargé, avec quelques modifications appropriées (voir [26] et les références qui y sont).

Dans le contexte des communications sans fil, les modèles de ”heavy traffic” ont été utilisés pour analyser les performances des algorithmes d 'ordonnancement ”maxWeight” et ”Exp” dans [27] et [28], respectivement. En outre, les auteurs de [29] ont étudié la performance d'une allocation de débit débit optimal dans un système avec plusieurs antennes pour deux utilisateurs (avec un canal commun pour les utilisateurs) lorsque le trafic entrant est presque égal au débit de service de chaque utilisateur. Cela a été généralisé dans [30] pour plusieurs antennes à l'émetteur, chacun desservant un utilisateur. Dans les deux cas, le comportement des files d'attente se révèle d'être un mouvement brownien multidimensionnel qui est contraint dans l'orthant positif. En outre, dans [31], il a été prouvé que pour le dernier cas, et les files d'attentes variables dans le temps, la politique d'attribution de taux à la limite de la région de capacité est asymptotiquement optimal dans le sens où elle minimise une somme pondérée des paquets dans la file d'attente lorsque la circulation est dense; cette quantité s'est avérée être un mouvement brownien qui reflète le changement de régime.

La première application de l'approche “heavy trafic” au problème de commande de puissance a été faite par [32], où il a été utilisé pour dériver une commande de puissance optimale dans le cadre d'une seule station de base desservant de nombreux utilisateurs mais via des canaux orthogonaux variant dans le temps. Dans ce travail, chaque utilisateur est préattribué un canal et les auteurs suppose des contraintes de puissance totale et que la puissance peut être réaffectée d'un canal à l'autre. La politique de contrôle a été spécifié numériquement. Les résultats de simulation de cette politique peuvent être trouvés dans [33]. En outre, dans [34] un contrôle optimal de puissance a été obtenu pour la liaison point-à-point sur un canal à évanouissement. Il a été montré que la politique de retard optimal est simple mono-seuil et des résultats de simulation montrent que le coût qui en résulte est très proche de celui obtenu par la résolution du problème de commande d'origine. Dans les trois derniers travaux, l'état du heavy trafic est imposé par la préallocation une quantité appropriée de puissance en fonction de l'état du canal et l'attribution d'un (beaucoup plus faible) montant ou une réserve de puissance selon des états de canaux et la longueur des files d'attente. En effet, sans cette allocation de puissance de réserve supplémentaire, le délai devient infini [34], [35].

Les principales contributions des travaux présentés dans ce chapitre sont: (*i*) Dérivation du modèle asymptotique pour un système de K liaisons sans fil interferentes pour les émetteurs équipés d'antennes uniques et multiples, (*ii*) Dérivation de une puissance de forme fermée (et précodage dans le cas des émetteurs à antennes multiples) contrôler la politique en vertu du canal parfait et la file d'attente des informations d'état de sorte que les objectifs (ref obj) soient atteints et (*iii*) Modification des algorithmes obtenus dans les modèles d'information moins exigeants.

Les travaux réalisés dans cette partie de la thèse ont été présentés dans deux congrés internationaux, une workshop interne d' Alcatel Lucent Bell Labs et une revue international:

(J1) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, A. Feki, "On Queue-Aware Power Control in Interfering Wireless Links: Heavy Traffic Asymptotic Modelling and Application in QoS Provisioning",*IEEE Transactions on Mobile Computing*, 2014, accepte

(C1) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, A. Feki, "Heavy Traffic Asymptotic Approach for Video Streaming over Small Cell Networks with Imperfect State Information", *28th Meeting of the Wireless World Research Forum*, Athens, Greece, Apr. 2012

(C2) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, A. Feki, "A Heavy Traffic

Approach for Queue-Aware Power Control in Interfering Wireless Links", *Proc. 13th IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, Cesme, Turkey, 2012

(W1) A. Destounis, B. Sayadi, A. Feki, M. Assaad, M. Debbah, "A Queue-Aware Power Control Policy in Dense Wireless Networks with Heavy Traffic: An Asymptotic Approach", *2nd Bell Labs Science Workshop*, Villarceaux, 6-7 December 2011

Sélection des Utilisateurs Actifs dans les systemes MISO multi-utilisateurs

L'utilisation de plusieurs antennes [4] a émergé comme l'une des technologies permettant d'accroître la performance des systèmes sans fil. La capacité de servir plusieurs utilisateurs dans le même bloc temps-fréquence a fait de l'utilisation de plusieurs antennes à la Station de Base (BS) une technique particulièrement intéressante pour les systèmes de liaison descendante multi-utilisateurs, et les avantages à venir de ce fait sont bien entendus [5]. Dans le détail, dans un système de liaison descendante où la BS a N antennes, et la plupart des N utilisateurs peuvent être programmées simultanément. Les décisions à prendre à chaque intervalle de temps sont alors (i) les utilisateurs qui devraient être ordonnés et (ii) la façon dont les signaux correspondants devraient être précodé .

D'un point du vue théorie de l'information, la région de la capacité du Canal de diffusion MIMO est bien caractérisé [36], en supposant avoir une connaissance parfaite des informations sur l'état de canal à l'émetteur. Cependant, la réalisation de cette région nécessite l'utilisation de "Dirty Paper Coding", qui est complexe à mettre en œuvre, alors que les hypothèses de parfaite informations d'état de canal et l'utilisation de dictionnaires gaussiennes. Ces hypothèses sont fortes dans les systèmes pratiques. Des techniques de précodage linéaires tels que Zero Forcing (ZF), une methode qui annule les interférences entre les utilisateurs, sont plus souhaitable à utiliser dans la pratique. Il existe de nombreux ouvrages sur la question de l' imparfait CSI, voir par exemple [37] et les références qui y sont, [38, 39, 40], mais elles se sont focalisées pricipalement sur des quantités comme le débit total et elles ne prennent pas en compte le processus de trafic des utilisateurs.

Ill est donc intéressant d'étudier l'impact de l'utilisation des plusieurs antennes sur la performance des couches supérieures [41]. Pour le canal d'accés multiple MIMO, une stratégie de pré-codage qui atteint la région de stabilité est présentée dans [42], selon les hypothèses de parfaite connaissance de l' état du canal et de la signalisation Gausienne. Cette politique est basée sur la minimisation de la dérivé de la fonction Lyapunov quadratique, étant donné les longueurs des file d'attente des canaux à chaque intervalle de temps et en faisant l'usage de codage en superposition et de décodage successif. Ceci est difficile à mettre en œuvre dans la pratique. En ce qui concerne la Canal de Diffusion (le système de liaison descendante), les auteurs de [43] ont proposé une technique basée sur précodage Zero Forcing (ZF), avec une politique d'ordonnancement des utilisateurs heuristique qui sélectionne les utilisateurs dont les canaux sont des vecteurs presque orthogonaux et illustrer la région de stabilité par simulations. Les auteurs de [44] prouvent que la politique résultante de la réduction de la dérivé d'une fonction de Lyapunov est de résoudre un problème de maximisation du taux de somme pondérées (avec des poids étant la longueur de la file d'attente) chaque intervalle de temps et ils proposent un algorithme du type "waterfilling" à cet effet. En outre, les auteurs de [45] proposent d'utiliser les retards des paquets dans la tête de chaque file d'attente comme les poids dans la fonction objective. Tous ces travaux supposent une connaissance précise du CSI est disponible à l'émetteur. Dans le cas d'informations retardées d'état de canal et canaux ayant une corrélation temporelle, les auteurs de [46] comparent la stabilité et les performances de retard opportuniste de formation de faisceaux et de l'espace temps de codage tout en [47] proposant un algorithme d'ordonnancement et precodage. En outre, dans [48], les auteurs étudient l'impact de la quantification de l' état du canal sur la stabilité du système où le precodage ZF est utilisé. L'ordonnancement des utilisateurs est fait de maniere centralsé, où l'emitteur sélectionne les utilisateurs en fonction des longueurs de files d'attente. Cependant, le fait que les ressources radio -par exemple le temps et/ou spectre sont nécessaires pour la SB pour obtenir des informations d'état du canal n'est pas pris en compte dans ces travaux.

Dans ce chapitre, nous considérons un système de liaison descendante MISO où la BS acquiert le CSI des utilisateurs en mode TDD, afin d'exploiter la réciprocité du canal. Il ya deux façons pour cela : (i) les utilisateurs estiment leur état du canal actuelle et transmettent le CSI de manière TDMA et (ii) les utilisateurs envoient des séquences de formation dans la liaison montante de sorte que la SB puisse estimer les canaux (les canaux de liaison montantes et de liaison descendantes sont les mêmes en raison de la réciprocité). Ce dernier système est mis en œuvre en utilisant des séquences orthogonales entre les utilisateurs, afin que la SB puisse décoder chaque transmission sans erreur. La longueur de ces séquences doit être proportionnelle au nombre d'utilisateurs qui forment en même temps dans la liaison montante. La formation de la liaison montante est considérée comme la stratégie la plus prometteuse pour les systèmes MIMO, puisque la longueur des séquences de formation ne dépend pas du nombre d'antennes à la SB. Toutefois, en raison de l'exigence d'orthogonalité, leur longueur est proportionnelle au nombre d'utilisateurs qui exécutent la formation de la liaison montante. Cela signifie que dans un système avec beaucoup d'utilisateurs, tous les utilisateurs devraient pas être choisis pour former en même temps, donc les utilisateurs qui devront former à chaque emplacement doivent également être sélectionnés. Le modèle de système de la formation en TDD de liaison montante a également été examiné dans [49], mais ils ne prennent pas en compte cette dernière observation. Dans cette partie de la thèse, nous nous concentrons sur le compromis entre avoir une formation de nombreux utilisateurs (afin d'avoir des données transmises aux nombreux utilisateurs en même temps) et ayant beaucoup de temps de la fente dédiée à la transmission de données (ce qui signifie avoir peu d'utilisateurs actifs). Pour simplifier les modèles, nous nous concentrons sur le cas ou l'émetteur utilise du précodage ZF. En outre, nous supposerons que tous les utilisateurs qui effectuent une formation de liaison montante dans une fente sont servis. Il s'agit d'une hypothèse utilisée assez souvent dans la littérature concernant les canaux de diffusion MIMO; dans ce contexte, la SB doit sélectionner l'ensemble des " utilisateurs actifs " à chaque intervalle de temps.

Une approche naturelle serait de laisser la SB seule décider quels utilisateurs servir dans chaque emplacement. C'est l'approche utilisée dans [48] et des normes en vigueur (par exemple LTE [11]), où la SB demande explicitement à certains utilisateurs leur CSI . Dans le cadre où les processus trafic/files d'attente sont considérés, l'ordonnancement des utilisateurs peut être fait sur la base des statistiques des canaux et de leur état de la longueur de la file d'attente à chaque emplacement. Malheureusement certains utilisateurs peuvent avoir des pauvres états de canal actuelles et certains utilisateurs avec de bons cannaux ne peuvent pas être ordonnés, ce qui réduit les performances du système. D'autre part, chaque utilisateur connait son état du canal actuel, et donc les politiques de rétroaction décentralisés où les utilisateurs décident en fonction de leurs états de canal actuels et peuvent améliorer les performances du système. Cela doit être fait correctement comme les politiques décentralisées ont besoin d'informations de signalisation supplémentaire qui peut diminuer considérablement la performance.

Il est à noter que il y a des travaux récentes [50], [51] qui ont montré que, dans un réseau avec une couche physique simple, des algorithmes décentralisés comme la récemment proposé "fast CSMA" [52] peut obtenir de bonnes performances. En outre, il a été montré dans des travaux antérieurs [53, 54] que des informations mettant à jour l'état du canal, qui est connu aux récepteurs, est plus important que l'information de longueur de file d'attente précise, au moins autant que la stabilité est concerné. Le scénario envisagé dans le présent chapitre est plus compliquée par rapport aux travaux récents sur l' ordonnancement décentralisé. En fait, dans les problèmes d'ordonnancement classiques (par exemple OFDMA ou TDMA), un utilisateur peut directement estimer son débit en utilisant l'état actuel du canal. Par contre, dans les systèmes MIMO multi- utilisateur, le débit de chaque utilisateur dépend des états de canal de tous les utilisateurs et alors l'utilisateur ne peut pas simplement estimer son débit binaire en utilisant l'état actuel du canal, ce qui complique fortement l'analyse.

Dans ce chapitre de la thèse, nous examinons trois approches sur le problème de sélection de l'utilisateur. La première est une approche centralisée, dans le sens que l'émetteur décide quels utilisateurs seront actives à chaque emplacement. La seconde approche, que nous appelons comme "décentralisée", est de laisser les utilisateurs décider lesquels entre eux devraient être actifs. Plus précisément, dans ce cas, l'émetteur précise le nombre d'utilisateurs à planifier et permet aux utilisateurs de décider d'une manière décentralisée qui seront ceux qui seront effectivement se programmés dans la fente . Combiné avec certains (rares) informations de signalisation concernant les utilisateurs queue longueurs de la BS, nous montrons que la combinaison correctement les approches décentralisées et centralisées conduit à une région de stabilité plus réalisable que l'utilisation de l'approche centralisée seul .

Les épreuves pour les dérivations des régions de stabilité sont effectuées sur la base de la méthode de démontrer d'abord que la région indiquée est réalisable par une règle qui ne tient pas compte de la longueur des files d'attente, prouver, à l'aide du critère Foster- Lyapunov, que la politique proposée atteint au moins aussi grande région comme la première règle et prouver qu'il n'y a pas de politique qui peut fournir une région de stabilité plus grande que la région indiquée. Cette méthode a été utilisée pour la première fois dans [13] puis dans de nombreux autres ouvrages traitant des problèmes d'ordonnancement et de la stabilité. Voir, par exemple , [?] et [55] ,[56], où les asymptotiques fluide du système sont examinés .

Les traveaux de ce chapitre ont donné lieu à une publication acceptée et une publication en cours de soumission dans les congrès internationaux

(C5) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, "Traffic-Aware Training and Scheduling for the 2-user MISO Broadcast Channel", *Proc. IEEE International Symposeum on Information Theory (ISIT)*, Honolulu, HI, USA, 2014

(C6) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, "A Dynamic Threshold-Based Approach for Active User Selection in Wireless MISO Downlink Systems", en cours de soumission

Une publication journal basée sur les traveaux et résultats du chapitre est egalement soumise

(J2) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, "Traffic-Aware Training and Scheduling for Wireless MISO Downlink Systems", *IEEE Transactions on Information Theory*, soumise

Feedback et Ordonnancement des Utilisateurs

Dans ce chapitre, nous abordons le problème de la rétroaction et l'ordonnancement dans les systèmes de liaison descendante multi-utilisateurs utilisant des canaux parallèles pour servir les utilisateurs. Ce paramètre correspond à des systèmes simples cellules de OFDMA et/ou les systèmes avec plusieurs antennes aux stations de base où la formation des faisceaux orthonormale est utilisée. Les deux régimes sont effectivement mis en œuvre dans les normes LTE [11] et peuvent offrir une augmentation substantielle de la performance du système. Afin d'exploiter pleinement le potentiel de ces techniques, la connaissance des états de canaux des utilisateurs est nécessaire. Cependant, à la fin, un seul utilisateur sera prévu dans chaque canal. Partant de cette observation, d'un système à un seul canal où l'objectif est de maximiser l'efficacité spectrale, les auteurs de [57] montrent que ce n'est pas vraiment nécessaire que tous les utilisateurs signallent leurs états des canaux, et ils proposent une politique basée sur un seuil qui peut réduire le montant de rétroaction en atteignant tous les avantages de la diversité multi-utilisateur. D'autres idées pour limiter le montant de rétroaction requis comprennent du regroupement des sous-porteuses et envoi d'un seul CQI (indicateur de la qualité du canal) à ces derniers [58] et/ou déclarer les canaux les plus forts de chaque utilisateur [58], [59]. En outre, certains régimes fondés sur l'accès aléatoire en utilisant un canal de collision pour les informations, par exemple [60], [61], [62] ont été proposés, en réglant les probabilités de transmission et les seuils d'une manière appropriée. Cependant, l'objectif principal de ces travaux est le taux d'efficacité spectrale / somme et ils ne prennent pas en compte les processus de circulation entrantes des utilisateurs .

En ce qui concerne l'effet de rétroaction sur le rendement de la stabilité, les auteurs de [56] ont étudié le problème de décision qui permet de définir les sous-ensemble d'utilisateurs pour acquérir leur information du canal, tandis que les auteurs de [54] ont enquêté sur la région de stabilité possible dans un système multi-canal avec mesures de canal peu fréquents. Dans ces travaux les statistiques des canaux sont supposées connues. En outre, dans [52], une stratégie à la base de CSMA est présentée pour transmettre les informations d'état de canal et dans [63] les auteurs élaborent un système de rétroaction pour une liaison descendante MIMO multi-utilisateur en utilisant la formation de faisceau orthonormale. Dans ces cas, toutefois, les auteurs ne prennent pas en compte le fait que la station de base doit attendre une certaine durée dans la fente avant de pouvoir utiliser la rétroaction. En supposant les statistiques du canal connus, les auteurs de [64] proposent un système de rétroaction heuristique avec deux fentes de rétroaction fondées sur l'idée de planification maximale quantile. En plus, dans [65] il est démontré que pour un système de L transporteurs avec le mode FDD pour les informations, la station de base doit acquérir au moins $\Theta(L)$ réalisations de canal dans chaque intervalle de temps afin d'obtenir de très près de la plus grande région de stabilité possible. Dans [66], un mode de rétroaction TDD est utilisé : la station de base demande aux utilisateurs d'envoyer les états de leur canal mais chaque procédure est centralisée et prend une partie de la tranche de temps. Sur la base de la théorie de l' arrêt optimal et en supposant que les distributions des gains de canal sont connus dans la station de base, les auteurs dérivent les propriétés générales de la politique centralisée de palpage optimale et le caractérisent complètement dans certains cas particuliers. Enfin , pour le même modèle, les auteurs de [67, 68] ont récemment proposé un schéma simple de rétroaction pour un système à un seul canal. Ce système, appelé Ordonnancement et Rétroaction sélective (SFF), fixe comme seuil le taux d'utilisateur avec une longueur maximale de la file d'attente et ne nécessitant aucune connaissance de canal et statistique du trafic. Il est montré à garantir une plus grande région de la stabilité d'un régime où tous les canaux sont sondés . Dans les systèmes multi-porteuses , le problème de palpage est plus difficile car un utilisateur peut être prévue sur un sous-ensemble de canaux et donc chaque utilisateur a besoin pour nourrir le dos CQI d'un sous-ensemble (aussi petit que possible) de ses chaînes . Appliquant directement les régimes mentionnés ci-dessus pour les systèmes multi-porteuses peut pas aboutir à une bonne région de stabilité parce que le nombre d'utilisateurs d'alimentation de retour sur chaque canal peut être encore grande . Cela pose un problème plus que le nombre d'utilisateurs dans la cellule augmente .

Dans ce chapitre, nous nous concentrons sur la liaison descendante d'un système multicanal de la cellule unique avec retour en mode TDD. En fait, dans le partie précédant nous avons déjà montré que si on suppose qu'un système de contention peut être réalisée à temps continu avec des signaux de contention très courts, une très grande partie de la région de la stabilité de l'idéal peut être obtenue. Cependant, ces hypothèses peuvent être assez fortes en pratique. In cette partie de la thèse, nous proposons deux politiques à la fois de qui un seuil pour le taux réalisable du canal est ajusté par la station de base en fonction des longueurs de file d'attente des utilisateurs, dans un manière similaire à [68]. Nous examinons deux approches : dans la première, un utilisateur dont le taux réalisable dépasse le seuil nourrit de retour avec une probabilité bien définie et dans le second de chaque utilisateur dont le taux réalisable est au-dessus du seuil alimente en arrière, mais la station de base peut décider quand arrêter la procédure de retour . Nous illustrons à la fois par des simulations et des analyses que ces systèmes surpassent le régime de SSF de [68] en termes de région de stabilité possible.

Les traveaux de ce chapitre ont donné lieu à deux publications dans des congrès internationaux:

(C3) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, "A Randomized Probing Scheme for Increasing the Stability Region of Multicarrier Systems", *Proc. IEEE International Symposeum on Information Theory (ISIT)*, Istanbul, Turkey, 2013

(C4) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, "A traffic aware joint CQI feedback and scheduling scheme for multichannel downlink systems in TDD feedback mode", *Proc. 24rth IEEE International Symposeum on Personal, indoor and Mobile Radio Communications (PIMRC)*, London, United Kingdom, 2013

En outre, ils ont lieu à deux applications des brevets Européennes par Alcatel-Lucent:

(P1) A. Destounis, B. Sayadi, M. Debbah, M. Assaad, "Method and device for transmitting buffered data from a base station of a wireless communication network to user equipments", 2013, European Patent, 813576, filed 12/04/2013, Alcatel-Lucent

(P2) A. Destounis, B. Sayadi, M. Debbah, M. Assaad, "Control of a Downlink

Radio Frequency Transmission from a Multi-Channel base Station", 2013, European Patent, 13305852.9, filed 21/06/2013, Alcatel-Lucent

Conclusions et Perspectives

En ce qui concerne le canal d'interférence, nous avons démontré que la modélisation asymptotique "heavy traffic" peut être un outil pour obtenir des directives utiles pour obtenir des algorithmes de contrôle de précodage et puissance. Une extension immédiate de ces travaux est le cas où les états des canaux sont reçus avec des erreurs à chaque émetteur: si les statistiques des erreurs sont connus , nous pouvons arriver à un modèle similaire où la moyenne sera également être donné au cours des processus d'estimation de canal (ce qui peut nécessiter des calculs plus lourdes dans le modèle de système). En outre, le cas où chaque émetteur sert plusieurs récepteurs dans une manière TDMA, où la séquence des utilisateurs qui sont servis dans chaque fente est pré- spécifié et reste constante (ou change très lentement) pendant le fonctionnement du système pourra être traité en modifiant légèrement notre modèle. Cependant, étendre davantage les résultats en utilisant ordonnancement opportuniste et/ou en fonction de file d'attente n'est pas simple et l'analyse doit être plus élaborée. Une autre extension possible serait de trouver le contrôle de précodage qui minimise la puissance afin d'atteindre les contraintes de dépassement de capacité des file d'attente. Le cours de l'action ici serait de (i) trouver la répartition de l'équilibre avec la puissance moyenne minimale et (ii) résoudre le problème du contrôle de l' équation différentielle stochastique en essayant de minimiser la puissance de réserve tout en satisfaisant les contraintes. La théorème de convergence pour la modèle asymptotique qu' on a prouvé dans cette thèse est valide aussi dans ce cas, mais avec des expressions plus complexes. Le problème de contrôle résultant peut être soluble que numériquement, comme le découplage de l' équation différentielle stochastique multidimensionnelle ne sera pas possible. Enfin, on pourrait utiliser des files d'attente dans les récepteurs à la place des émetteurs, afin de modéliser le streaming vidéo, et d'utiliser une approche asymptotique similaire pour la contrôle de precodage. Un travail récent [69] est dans cet esprit .

La conclusion principale des travaux concernant la sélection des utilisateurs actifs dans les systèmes de diffusion MISO et le feedback et ordonnancement des utilisateurs est que les utilisateurs peuvent connaitre leur état de canal instantané, qui ensuite peut être utilisé dans les systèmes de communications radio futurs pour en tirer profit. Plus précisément, nous avons montré dans le chapitre 4 que, en supposant que les procédures de contention idéalisées, à l'aide d'une politique où la station de base signale le nombre d'utilisateurs d'être sélectionné et les utilisateurs décident de manière décentralisée à base de conflit qui va participer à l'ordonnance peut agrandir la zone de stabilité d'un système MISO par rapport à la sélection des utilisateurs centralisée. En outre, pour une antenne unique à l'émetteur, une politique décentralisée basée sur la contention peut atteindre une grande fraction de la région de stabilité du système idéal (c'est-a-dire la région du stabilité d'un système dans le cas ou toutes les réalisations des canaux des utilisateurs sont connues a l'émetteur sans cout). Même dans les cas où minislots doivent être utilisés pour le feedback, la connaissance des utilisateurs de leurs états de canal peut être exploité par une stratégie où l'émetteur choisi des seuils dynamiques pour les états des gain de canaux. Ces seuils sont une fonction de l'état des fils d'attente des utilisateurs. En plus de l'interprétation que le système peut prendre en charge plus de la demande de trafic en temps non réel, l'extension de la zone de stabilité implique la réalisation de plus grande utilité si une approche d'optimisation fondée sur l'utilité doit être utilisé. Plus précisément, définir une fonction d'utilité

$$U(\bar{\mathbf{r}}) = \sum_{k=1}^{K} U_k(\bar{r}_k),$$

ou $\bar{r}_k$ est est la moyenne à long terme de taux de l'utilisateur k et $U_k(x)$ est une fonction concave qui est connue a l'émetteur. Dans ce cas, on démontrer que le vecteur optimal de taux à long terme réside dans la limite de la zone de stabilité, voir par exemple [15], donc l'élargissement de la zone de stabilité conduit à plus grande utilité optimale. En outre, au moins pour les systèmes de liaison descendante avec une cellule unique, un procédé général de l' optimisation d'utilité consiste à créer des files d'attente virtuelles $q_k(t)$ avec processus d'arrivée contrôlée de sorte que

$$\mathbf{a}(t) = \arg \max_{\mathbf{x} \in [0,A]^K} \left\{ VU(\mathbf{x}) - \sum_{k=1}^{K} x_k q_k(t) \right\},$$

ou $V > 0$ est une constante. L'ordonnancement doit être telle que ces files d'attente virtuelles sont stables. On peut alors montrer que l'utilité est atteint dans $O(1/V)$ de l' optimal, et la somme moyenne des longueurs des files d'attente sont $O(V)$ [15]. Cette théorie générale a été appliquée pour l'ordonnancement des utilisateurs et l'allocation des ressources dans les réseaux sans fil avec des résultats prometteurs, voir par exemple [70], [16], [17]. En ce qui concerne nos résultats, nous pouvons appliquer les mêmes techniques proposées dans les chapitres 4 et 5 utilisant les files d'attente virtuelles. En outre, sur les travaux concernant les systèmes de diffusion MISO, on peut utiliser le cadre général de l'optimisation des réseaux de [15] et considérer l'aspect de l'énergie consommée par les utilisateurs pour l'estimation des canaux dans la station de base, et minimiser cette énergie avec la contrainte du stabilité du système, ou examiner le problème de la stabilité/maximisation de l'utilité sous contraintes sur la puissance moyenne dépensés pour l'estimation. Une motivation pour ce genre de considérations est de prolonger la durée de vie de la batterie des utilisateurs. Comme la connaissance du canal de l'État devient encore plus important dans ce cas, notre intuition est que les politiques de sélection d'utilisateurs décentralisés aient des performance encore meilleures que celles centralisées. Extension des résultats de la thèse long de ces lignes est l'objet de travaux en cours:

(J3) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, "On User Selection for Network Utility Optimization in Wireless MISO Downlink Systems",en cours de préparation

Travaux Futurs

Les résultats de la présente thèse concernent essentiellement le comportement des files d'attente dans le systèmes sans fil. Toutefois, une mesure plus importante dans la pratique est le retard/temps subi par les données en attente dans les files d'attente jusqu'à ce qu'ils soient transmis. Alors que des retards et des longueurs de file d'attente sont des quantités liées (par exemple, le délai moyen est égal à la longueur de la file d'attente moyenne divisé par le taux d'arrivée par la loi de Little et intuitivement on peut supposer que les paquets dans une longue file d'attente connaîtront des retards importants), l'analyse des délais exacte est connu pour être beaucoup plus difficile que l'analyse de la longueur de la file d'attente. Il serait intéressant de voir comment les algorithmes d'ordonnancement à base de retard effectuent, par exemple, dans l'esprit des travaux récents [71]. D'un autre coté, on peut regarder dans le régime des grandes déviations afin d'obtenir plus d'informations sur le comportement du système. Ce genre de comportement asymptotique a été aussi largement utilisé dans l'analyse d'algorithmes d'ordonnancement dans les systèmes de files d'attente en général et dans les systèmes sans fil en particulier (voir par exemple [72, 73] pour le second), et les travaux récents [74] et [63] étudient la performance des algorithmes d'ordonnancement et rétroaction de ce régime. De plus, on peut combiner ces approches avec la théorie de la bande passante efficace et la capacité effective [75] pour obtenir une certaine estimation des distributions de retard dans les fils d'attente.

Les approches ci-dessus et les schémas proposés dans cette thèse sont contraint par la suppression des données dans les files d'attente. Cependant, la plupart des applications peuvent tolérer une certaine perte de données, mais ont des contraintes strictes sur les délais. Les travaux récents sur ce sujet sont notamment [76], [77] pour les réseaux sans fil avec une couche physique relativement simple et [78] pour les réseaux câblés avec un routage fixe. Il serait intéressant d'examiner les problèmes de ce genre dans les réseaux à petites cellules et/ou des systèmes MIMO massives. Dans le premier, les théories comme l'apprentissage de la machine ou des systèmes de contrôle décentralisés/réseau peut être essentiel pour surmonter la backhaul de contrôle limité reliant les cellules. Dans le dernier, car il y aura de nombreux utilisateurs dans la cellule , la sélection de l'utilisateur est un problème important (les durées de tranche de temps et des temps de cohérence sont donnés) et il sera intéressant de voir si on peut tirer utiliser les résultats de cette thèse dans le cas ou les utilisateurs sont sensibles aux délais.

Enfin, un aspect qui n'est pas pris en compte dans cette thèse est le fait que, dans un réseau, les utilisateurs arrivent et partent, voir par exemple [79] et les références qui y sont. Par exemple, un utilisateur peut arriver dans le réseau, demander un fichier et partir quand ce transfert est achevé. L'ordonnancement et l'allocation de ressources dans ce cadre posent plus de problèmes, car les algorithmes du type "MaxWeight" ne sont plus optimaux [80] dans le cadre de stabilité du réseau. Ce réglage peut être appliqué pour fournir une évaluation de la performance robuste des approches récentes dans ordonnancement proactif [81] et la mise en cache dans les petites stations de base cellulaire [82], est proposée afin de faire un usage plus efficace de la liaison terrestre limitée. En outre, ces modèles avec les utilisateurs qui arrivent et partent dans le réseau peuvent être utilisés pour fournir des lignes directrices sur ce que les fichiers de prélecture, où et quand.

Contents

List of Figures

List of Tables

Notations

$2^{\mathcal{K}}$	Power set of set $\mathcal{K}$
$[x]^+$	$\max\{0, x\}$
$\Gamma(N)$	Gamma function with parameters N: $\Gamma(N) = \int_0^\infty t^{N-1}e^{-t}dt$
$\gamma(x;N)$	Lower incomplete Gamma function: $\gamma(x;N) = \int_0^x t^{N-1}e^{-t}dt$
$\lfloor x \rfloor^+$	Integer part pf x
$\mathbb{C}^K$	The space of K-dimensional vectors with complex entries
$\mathbb{E}$	Expectation operation
$\mathbb{P}$	Probability measure
$\mathbb{R}_+^K$	The space of K-dimensional vectors with nonnegative real entries
$\mathbb{Z}_+^K$	The space of K-dimensional vectors with nonnegative integer entries
$\mathbf{A}$	Matrix
$\mathbf{a}$	Column vector
$\mathbf{A}^T$	Complex conjugate transpose of matrix $\mathbf{A}$
$\mathbf{A}^T$	Transpose of matrix $\mathbf{A}$
$\mathbf{A}^\#$	Pseudoinverse of matrix, $\mathbf{A}$: $\mathbf{A}^H(\mathbf{A}\mathbf{A}^H)^{-1}$
$\mathbf{a}_j$	j-th column of matrix $\mathbf{A}$
$\mathbf{I}_N$	Identity matrix of size N
$\mathcal{CH}$	Convex hull

$\mathcal{CN}(\mu, \mathbf{R})$	Vector-valued complex normal random variable with mean μ and covariance matrix $\mathbf{R}$
$\mathcal{N}(\mu, \sigma^2)$	Real normal random variable with mean μ and variance σ^2
$\|\|\mathbf{a}\|\|$	Eucledian norm of vector $\mathbf{a}$
$\|\|\mathbf{a}\|\|_1$	L_1 norm of vector $\mathbf{a}$
$\stackrel{w}{\to}$	Weak convergence
$\mathrm{diag}\{x_1, ..., x_N\}$	Diagonal matrix with elements $x_1, .., x_N$ on its main diagonal
$\mathrm{tr}(\mathbf{A})$	Trace of matrix $\mathbf{A}$
a_{ij}	(i,j) Element of matrix $\mathbf{A}$
$B(a, b)$	Beta function with parameters a, b: $\int_0^1 t^{a-1}(1-t)^{b-1}dt$
$erfc(x)$	Complementary error function: $erfc(x) = \frac{2}{sqrt\pi} \int_x^\infty e^{-t^2}dt$
$f^{-1}(x)$	Inverse of the function $f(x)$:$f(f^{-1}(x)) = f^{-1}(f(x)) = x$
$I_B(x; a, b)$	Regularized incomplete Beta function: $I_B(x; a, b) = \frac{\int_0^x t^{a-1}(1-t)^{b-1}dt}{B(a,b)}$
a_k	k-th element of column vector $\mathbf{a}$
$I_{\{X\}}$	Indicator function

Acronyms

BC	Broadcast Channel
BS	Base Station
CDF	Cumulative Distribution Function
CQI	Channel Quality Indicator
CSI	Channel State Information
CSMA	Carrier Sense Multiple Access
FDD	Frequency-Division Duplexing
HJB	Hamilton-Jacobi-Bellman
HSDPA	High Speed Downlink Packet Access
i.i.d.	independent and identically distributed
ID	Identification Number
LTE	Long Term Evolution
MAC	Media Access Control
MDP	Markov Decision Process
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
OFDMA	Orthogonal Frequency-Division Multiplexing Access
p.d.f.	probability density function
QoS	Quality of Service
SC	Small Cell
SCN	Small Cell Network
SDE	Stochastic Differential Equation
SINR	Signal-to-Interference-plus-Noise Ratio
SISO	Single-Input Single-Output
SNR	Signal-to-Noise Ratio

TDD	Time-Division Duplexing
TDMA	Time-Division Multiple Access
VBR	Variable Bit-Rate
WiMax	Worldwide Interoperability for Microwave Access
ZF	Zero-Forcing

Chapter 1

Introduction

1.1 Motivation

Today's wireless and cellular networks experience an increase in the data demands of their users, and this is expected to be the trend for the next years. It is a result of more and more demand for data and video streaming applications, due to the proliferation of devices like smartphones and tablets. The following three ways of coping with this challenge are seen as the most prominent:

- **Small Cell Networks:** Small Cell Networks (SCNs) are dense wireless networks with cells of relatively small radius and low-power Base Stations (BSs) with aggressive frequency reuse [1]. The main idea is here to capitalize on the fact that network densification can provide better coverage and frequency reuse one can provide overall better capacity. On the other hand, having many BSs operating in the same frequency very close naturally calls for techniques for interference mitigation. Cooperation between the base stations of a cellular network has been shown to mitigate the intercell interference and increase spectral efficiency [2], [3]. However in practice this is not always feasible due to limitations in backhaul capacity. In addition, in practical cases user request some traffic from the core network, which arrives with random patterns, and they have requirements in terms of Quality of Service (QoS) (such as delays, data losses etc.). Thus, a good approach is to make the resource allocation at the base stations adapted to the real-time traffic characteristics and queue states. For example, a base station whose users have little or no data waiting to be sent can transmit with very low power for some time and avoid causing interference to neighbouring base stations with more pressing user demands.

This seems a more promising approach instead of focusing (only) on the traditional metrics of coverage and spectral efficiency.

- **Multiple antennas at the base stations:** The benefits of using multiple antennas in wireless systems are now very well understood, both in single- and multi- user systems [4, 5], mainly from a spectral efficiency point of view. In a SCN , the BS can use multiple antennas to manage interference more effectively [3]. In the single cell setting, the use of multiple antennas allows the BS to serve multiple users in the same time-frequency block, thus enhancing the system's performance. In order to fully realize the potential of multiuser Multiple-Input Multiple-Output (MIMO) , however, accurate knowledge of the channel states of the users is necessary. The most prominent way to do this is considered to be training, in Time-Division Duplexing (TDD) mode, by the users, meaning that the users send orthogonal sequences for the base station to estimate their channels. This technique exploits the channel reciprocity and has the benefit that the length of the training sequences is independent of the number of antennas. This observation has led to the promising concept of "Massive MIMO " , where the base stations have much more antennas than users [6, 7]. However, the length of the pilot sequences *does* scale proportionally with the number of active users. Since the coherence time (or the length of the slot used by the protocol) is limited, having many active users in a slot means that the time devoted to actually transmitting data is small. Therefore, user selection has to be done dynamically, taking into account this overhead and the traffic dynamics and current state of the buffers of each user.

- **Scheduling and use of parallel channels:** Exploiting the temporal variations of the fading channels (a concept called "multiuser diversity") to increase the spectral efficiency by selecting users with good channel conditions [8] is also very well understood in multiuser systems. This idea, referred to as "opportunistic scheduling", can provide high spectral efficiency and also fairness between the users [9], and has been used in 3G systems already [10]. Creating parallel channels in frequency, via Orthogonal Frequency-Division Multiplexing Access (OFDMA) modulations, or space, via the use of multiple antennas with orthonormal beamforming, can further enhance this concept. In fact both OFDMA and MIMO with orthonormal beamforming are parts of the latest Long Term Evolution (LTE) and LTE -Advanced standards for cellular communications [11].

Contrary to the other two cases (namely MIMO and reducing cell size), traffic/queueing- aware scheduling has been extensively studied [12] and remains a very active research topic, especially after the seminal work [13] introducing MaxWeight scheduling. However, most related works assume perfect Channel State Information (CSI) in the BS . This can be acquired only by feedback from the receivers. This needs time (in TDD systems) or frequency (in Frequency-Division Duplexing (FDD) systems) resources, which are limited and could have been used for transmission. The feedback overhead is even bigger in the case of systems with parallel channels, leading to an exploration-exploitation tradeoff between using resources to acquire information about many users (thus selecting users with good channels to transmit to) and having less users feeding back, thus reducing the overhead and using more resources for actual transmission.

Motivated by the above considerations, the focus of the thesis is on the interactions between the physical layer and Media Access Control (MAC) layer. More precisely, we study how physical layer resource allocation (power control, precoding, scheduling) and feedback overheads affect the MAC layer performance of the system. The latter is captured by the behaviour of the queues in the BSs where data for users are stored while waiting for transmission. Cross-layer radio resource allocation algorithms to improve this performance are proposed. Two performance measures are examined, namely (i) the probability that the queue lengths exceed a certain threshold and (ii)the stability region of the system, which in practice means the set of users' demands in terms of traffic arrival rates that the system can support while the users have bounded delays. The relevance of the first objective is to control the data loss from queue overflows and/or the delay resulting from queueing. The second objective is relevant for data users, since enlarging the stability region implies, in a sense, that the users can request downloads of a higher volume. On the other hand, the methods used for enlarging the stability region of the system can be also used in a straightforward manner for enhancing the performance of the system when the latter is measured as a function of the average rate per user [14, 15]. These methods have been employed in recent works [16, 17] concerning video streaming in wireless network with promising results.

1.2 Overview of the Manuscript

In Chapter 2 of the thesis, we present an overview of the main mathematical tools used. More concretely, a brief overview and basic results and concepts on heavy traffic asymptotic models and stochastic stability of queuing networks are given.

Chapter 3 deals with the problem of power control and precoding in transmitter-receiver pairs such that the probability of each queue length exceeding a specific threshold is fixed below a desired value. This model can correspond to satisfying QoS requirements of users with dynamic traffic arrivals in a network of small cells. Due to the complexity of the original problem, we use heavy traffic asymptotics to obtain an analytically tractable formulation. Informally, these are obtained by examining the network when the arrival rates are very close to the service rates. These kind of asymptotics have been extensively used in queueing theory for analysis and control of networks [26] and can give useful and relevant results and guidelines for practical systems, even in lower load conditions. One main contribution of Chapter 3 is the derivation of the Stochastic Differential Equation (SDE) that describes the network with the topology of the Multiple-Input Single-Output (MISO) or Single-Input Single-Output (SISO) interference channel in the heavy traffic limit. The other contribution is the use of this asymptotic model to propose algorithms and closed form expressions for a dynamic precoding and power control policy so that the QoS constraints are satisfied. Simulation results illustrate that, even obtained under some asymptotic model, these policies can indeed be effective in practice, also under the very relevant cases of each transmitter having access only to local Signal-to-Interference-plus-Noise Ratio (SINR) information and limited backhaul used for coordination between transmitters.

Chapter 4 deals with the problem of active user selection in MISO - OFDMA wireless systems such that the system's stability region is as big as possible (that is, the system can satisfy as many traffic demands as possible while keeping finite delays). We focus on a system in a rich scattering environment (Rayleigh fading) where Zero-Forcing (ZF) precoding is used to serve all active users in every slot and the training is used for the base station to acquire the channel state information of the users. We consider the centralized policy where the BS selects the users based on their queue lengths and their channel statistics, and a decentralized one where the BS periodically broadcasts the number of users to be scheduled and suitable quantized queue lengths of the users. In the latter, the users contend to get scheduled in a Carrier Sense Multiple Access (CSMA) - based manner, with waiting times calculated based on their channel realization and signalling from the BS . We show that combining these two policies (i.e. dynamically switching between them) can enlarge the stability region of the system with respect to the centralized policy alone. The stability regions are calculated for the general setting and illustrated in the case of the 2-user system. In addition, we show that, in the case of single antenna transmitter, using the decentralized policy alone can achieve a very big fraction of the region achievable in the ideal case with full CSI at no cost, without the need of knowing the channel statistics and for arbitrary channel distributions. The results here suggest that letting the users participate more actively in the schedule decision is something that should be considered in next generation systems.

Chapter 5 deals with the problem of joint feedback and scheduling in downlink systems employing parallel channels to serve the users. Contrary to the Section of Chapter 4 for the single - antenna BS , we do not make the assumption of continuous time model for contention. Channel Quality Indicator (CQI) feedback is done in TDD mode and the focus is in finding a balance between acquiring feedback from many users and exploiting current feedback to enlarge the stability region of the system as much as possible. Based on the idea that a threshold on the supported rate is set dynamically every time slot at each channel, we propose two schemes: one where a user decides at random with some properly defined probability to feed back or not and another where the BS decides when to stop the feedback procedure. These schemes are proven to outperform existing works and work well in cases where the users and BS do not have even statistical information of the channels. This chapter also suggests that the fact that the users know their channels at each slot should be further leveraged in future wireless system to make more effective use of the available resources.

Finally, Chapter 6 concludes the thesis and presents possible extensions of the results and future research directions.

1.3 List of publications

The main results obtained in the course of the thesis have been communicated in the form of publications in academic journals, proceedings of international conferences and a presentation in an internal company workshop. The full list is as follows:

Journal papers :

(J1) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, A. Feki, "On Queue-Aware Power Control in Interfering Wireless Links: Heavy Traffic Asymptotic Modelling and Application in QoS Provisioning",*IEEE Transactions on Mobile Computing*, 2014, to appear

(J2) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, "Traffic-Aware Training and Scheduling for Wireless MISO Downlink Systems", *IEEE Transactions on Information Theory*, submitted

(J3) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, "On User Selection for Network Utility Optimization in Wireless MISO Downlink Systems", in preparation

Conference papers :

(C1) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, A. Feki, "Heavy Traffic Asymptotic Approach for Video Streaming over Small Cell Networks with Imperfect State Information", *28th Meeting of the Wireless World Research Forum*, Athens, Greece, Apr. 2012

(C2) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, A. Feki, "A Heavy Traffic Approach for Queue-Aware Power Control in Interfering Wireless Links", *Proc. 13th IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, Cesme, Turkey, 2012

(C3) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, "A Randomized Probing Scheme for Increasing the Stability Region of Multicarrier Systems", *Proc. IEEE International Symposeum on Information Theory (ISIT)*, Istanbul, Turkey, 2013

(C4) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, "A traffic aware joint CQI feedback and scheduling scheme for multichannel downlink systems in TDD feedback mode", *Proc. 24rth IEEE International Symposeum on Personal, indoor and Mobile Radio Communications (PIMRC)*, London, United Kingdom, 2013

(C5) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, "Traffic-Aware Training and Scheduling for the 2-user MISO Broadcast Channel", *Proc. IEEE International Symposeum on Information Theory (ISIT)*, Honolulu, HI, USA, 2014 (accepted)

(C6) A. Destounis, M. Assaad, M. Debbah, B. Sayadi, "A Dynamic Threshold-Based Approach for Active User Selection in Wireless MISO Downlink Systems", in preparation

Workshop presentations:

(W1) A. Destounis, B. Sayadi, A. Feki, M. Assaad, M. Debbah, "A Queue-Aware Power Control Policy in Dense Wireless Networks with Heavy Traffic: An

Asymptotic Approach", *2nd Bell Labs Science Workshop*, Villarceaux, 6-7 December 2011

In addition it has led to the following patents that have been filed by Alcatel-Lucent:

(P1) A. Destounis, B. Sayadi, M. Debbah, M. Assaad, "Method and device for transmitting buffered data from a base station of a wireless communication network to user equipments", 2013, European Patent, 813576, filed 12/04/2013, Alcatel-Lucent

(P2) A. Destounis, B. Sayadi, M. Debbah, M. Assaad, "Control of a Downlink Radio Frequency Transmission from a Multi-Channel base Station", 2013, European Patent, 13305852.9, filed 21/06/2013, Alcatel-Lucent

Chapter 2

Mathematical background

This Chapter presents a short overview of key mathematical concepts and results that are used throughout the thesis. To begin with, define a discrete time system of K queues (users), with $q_k(t)$ the length of each queue at the beginning of timeslot t, $a_k(t)$ the corresponding traffic process (i.e. the amount of data coming into the queue at timeslot t) and $\mu_k(t)$ the service process, that is the amount of data that can get transmitted at timeslot t. Note that in the context of wireless communications, the latter depends on the state of the wireless channels at t and the scheduling and resource allocation algorithms employed. In addition, the real amount of data transmitted to user k at timeslot t is $\min\{q_k(t), \mu_k(t)\}$, since the service offered can be greater than the amount of data already in the queue (e.g. in the case of a user having a very good channel). For the rest of the thesis, queues will be measured in bits and arrival and service processes in bits per timeslot unless specified otherwise. In general, the arrival processes can be correlated across time but they are independent across users and ergodic, with a finite mean arrival rate λ_k and variance σ_k^2. Since this thesis is devoted to study the downlink of wireless systems, traffic to each queue is single hop, i.e. the data is transmitted from queue k directly to the intended receiver and no routing/relaying are examined.

2.1 Functional Limit Theorems

In this section we present some basic functional limit theorems, namely the functional law of large numbers and the functional central limit theorem. They can be seen as analogues of the law of large numbers and central limit theorem in stochastic processes. For the first we have:

Theorem 2.1.1 (Functional Law of Large Numbers)**.** *Assume* $x(\tau), \tau = 0, 1, 2, ...$ *is a sequence of i.i.d. random variables with mean* μ*. Then*

$$\frac{1}{n}\sum_{\tau=0}^{\lfloor nt \rfloor} x(\tau) \to \mu t$$

with probability 1*.*

An important process in the following is the Brownian Motion, or Wiener process. It is defined as follows:

Definition 2.1.1 (Standard Wiener process)**.** *A continuous time stochastic process* $w(t)$ *is called Standard Wiener process (or standard Brownian motion) if it satisfies the following conditions:*

- *Its sample paths are continuous with probability* 1
- $w(0) = 0$
- *Its increments are mutually independent*
- $w(t) - w(s) \sim \mathcal{N}(0, t-s)$, $\forall 0 \leq s < t < \infty$

For proof of existence and constructions of such a process refer to e.g.[83]. The Wiener process is used for the analogue of Central Limit Theorem in the case of stochastic processes:

Theorem 2.1.2 (Functional Central Limit Theorem)**.** *Assume* $x(\tau), \tau = 0, 1, 2, ...$ *is a sequence of i.i.d. random variables with mean* μ *and variance* σ^2*. Then* [1]

$$\frac{1}{\sqrt{n}}\sum_{\tau=0}^{\lfloor nt \rfloor} \frac{x(\tau) - \mu}{\sigma} \xrightarrow{w} w(t),$$

where $w(t)$ *is a standard Wiener process .*

The functional laws presented above and their extensions have been used to derive asymptotic models of queuing networks for performance evaluation and optimization, see e.g. [84]. More specifically, application of the Functional Law of Large Numbers usually leads to an asymptotic model that depends on the first order statistics of the system (i.e. mean arrival and service rates). In this case an Ordinary Differential Equation describes the evolution of the queue lengths. This method has been heavily used for stability analysis of networks, beginning with the work in [85] and applied to wireless networks with scheduling in many works, e.g. [55], [53]. In addition, control policies for the network can

[1]The notation $\xrightarrow{w}$ denotes weak convergence

be derived from such models [86]. On the other hand, asymptotics based on the Functional Central Limit Theorem lead to the evolutions of the queue lengths being described by SDEs [83, 87], constrained in the nonnegative orthant. The advantage of these models is that, in addition to the mean behaviour of the system, they capture its stochastic behaviour as well, keeping the second order statistics of the arrival and service (and routing if applicable) processes. The usual interpretation of these models is that they describe a network where the arrival rates at the queues [2] are pushed very close to the corresponding service rates, with the gap closing down as $O(1/\sqrt{n})$ [3], where $n \to \infty$ is the scaling parameter in the Functional Central Limit Theorem. Due to this interpretation, they are often referred to as "Heavy Traffic Approximations". This kind of asymptotic models has been extensively used mainly for performance evaluation of queuing networks due to its ability to capture the stochastic behaviour while actually simplifying the system model; in practice it is shown that the asymptotic models describe the original systems rather accurately, even in the case where the system is not very heavily loaded.

2.2 Stability of Queuing Systems

Stability, which is the focus of Chapters 4 and 5, is a very important aspect of queuing systems. Formally, its definition is as follows:

Definition 2.2.1 (Strong Stability). *A system is said to be strongly stable if*

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{q_k(t)\} < \infty, \forall k \in \{1,..,K\}$$

Intuitively stability implies that the mean queue length of every queue in the system is finite, further implying finite delays in single hop systems. The figure of interest in this thesis is strong stability, therefore in the remainder of the manuscript "stable" will imply "strongly stable" unless stated otherwise. If the arrivals and service rate processes are such that the Markov chain is irreducible and aperiodic with a single communicating class, strong stability is equivalent to positive recurrence of the chain [15].

The above definition holds for a fixed mean arrival rates and resource allocation policy, and leads to the concept of a stability region.

[2] Or at the queues with the heaviest load in terms of multi-hop networks

[3] Another similar interpretation is that the load of the queue grows as $\rho = 1 - b/\sqrt{n}$ for a constant $b > 0$

Definition 2.2.2 (Stability Region)**.** *The stability region Λ of a resource allocation policy is defined as the set of vectors of mean arrival rates for which the system is stable under this policy. Furthermore, an algorithm that achieves the maximum possible stability region is called throughput optimal.*

For the rest of the thesis, when describing stability regions we will mean that the system is stable in the *interior* of the calculated region. The behaviour on the boundary is not examined - usually for the boundary points the system is stable in at least a weaker sense, i.e. mean rate stable [15]. In short, and rather informally, a system is stable when the mean service rate of each user is bigger than the mean arrival rate of the corresponding traffic process. In the case of full channel knowledge, it is known that a throughput optimal algorithm is the so-called MaxWeight [13], which, in the case of a single-hop system, maximizes the quantity $\sum_{k=1}^{K} q_k(t)\mu_k(t)$ (see also [88]). Other throughput optimal algorithms can be obtained using the same concept but with different weights (appropriate functions of the queue lengths and/or delays experiences by the data unit at the head of the queues) in each service rate [12, 55, 27, 28, 53, 89].

One basic tool used to prove stability of a system comes from the general theory of stability of stochastic systems [90] and it is based on methods including Lyapunov functions. The following definitions are in order:

Definition 2.2.3 (Lyapunov function)**.** *A function $V : \mathbb{R}^K \to \mathbb{R}$ is said to be a Lyapunov function if it has the following properties*

- $V(\mathbf{x}) \geq 0, \forall \mathbf{x} \in \mathbb{R}^K$
- *It is non-decreasing in any of its arguments*
- $V(\mathbf{x}) \to +\infty, as\, ||\mathbf{x}|| \to +\infty$

Definition 2.2.4 (Lyapunov drift)**.** *The (one-step) drift of a Lyapunov function V for the system $\mathbf{q}(t)$ is defined as*

$$\Delta V(\mathbf{x}) = \mathbb{E}\left\{V(\mathbf{q}(t+1)) - V(\mathbf{q}(t))|\mathbf{q}(t) = \mathbf{x}\right\}.$$

In our context, the expectation is with respect to the arrival processes, channel processes and possible randomizations of the resource allocation algorithms (which affect the service processes). A main result of stochastic stability theory is the so-called Foster-Lyapunov criterion which connects the stability of the system with the drift of a Lyapunov function:

Theorem 2.2.1 (Foster-Lyapunov criterion)**.** *Assume there exists a Lyapunov* *function* $V(\mathbf{x})$ *and a bounded set* $\mathcal{B} \subset \mathbb{R}_+^K$ *such that*

$$\Delta V(\mathbf{x}) < \infty, \forall \mathbf{x} \in \mathcal{B},$$
$$\Delta V(\mathbf{x}) < 0, \forall \mathbf{x} \notin \mathcal{B}.$$

Then the system is stable.

A Lyapunov function often used in practice is the quadratic function $V(\mathbf{x}) = \frac{1}{2}\sum_{k=1}^{K} x_k^2$ [4]. In this case another sufficient condition for stability, which follows from the Foster-Lyapunov criterion, is the following:

Theorem 2.2.2. *Set* $V(\mathbf{x}) = \frac{1}{2}\sum_{k=1}^{K} x_k^2$. *If there exist constants* $B < +\infty$ *and* $\epsilon > 0$ *such that*

$$\Delta V(\mathbf{q}(t)) \leq B - \epsilon \sum_{k=1}^{K} q_k(t),$$

then the system $\mathbf{q}(t)$ *is stable.*

This sufficient condition was in fact used to prove stability of the MaxWeight algorithm in [13]. It is also widely used in more general optimization and control problems in wireless networks, starting with the works of [91] and [14]. The reader is referred to [15] for a comprehensive treatment of the techniques based on Lyapunov functions for stochastic network optimization.

[4] Or $V(\mathbf{x}) = \sum_{k=1}^{K} x_k^2$, but the results are the same

Chapter 3

Queue-Aware Power Control and Precoding for the MISO Interference Channel

3.1 Introduction

In this Chapter we consider a system with K transmitters, operating in the same frequency band W, each having L antennas and serving one receiver. The challenge here is that links interfere with each other so the queue length of one transmitter depends also on the power (and precoder) allocations of the other transmitters. This setting can correspond to Small Cells (SCs) employing the same carrier frequency to serve the users (and in a more general cellular networks context where each base station serves one user in a given frequency). The objective is to minimize the total power over an infinite time horizon such that the probability that the queue size at each transmitter exceeding a threshold is fixed. This goal may correspond to fixing a data loss probability (for finite buffers at the transmitters) in some desired values or some delay requirement, which is a crucial QoS aspect in multimedia applications. In delay-constrained cases the (average) delay is proportional to the queue length due to Little's law. Therefore we can argue that bounding the delay below a desired threshold is in a sense equivalent to bounding the queue length below a threshold corresponding to the delay bound. Since the incoming traffic and channels evolve randomly however, such a constraint is not possible to hold. We will thus consider the following probabilistic QoS metric for the queue length at each transmitter k

$$\mathbb{P}\left\{q_k(t) > q_k^{thr}\right\} = \delta_k, \tag{3.1}$$

which means setting a buffer outage probability in some values that can be tolerated by the application (this also implies that the delay will be small enough most of the time). We tackle the problem by proposing a power control strategy so that these constraints are met, based on heavy traffic asymptotic modelling. The approach is then to divide the power into two parts: equilibrium and reserve powers. The equilibrium power problem consists in allocating the power according to the channel states so that on average the transmission rate is equal to the arrival rate. In the literature, this type of problems has been widely studied for both single and multiple antennas systems (one can refer to [18],[19] for more details). The main challenge lies in the modelling and allocation of the reserve power. An immediate approach to optimally control the reserve power would be to formulate the problem using optimal control theory and Hamilton-Jacobi-Bellman (HJB) framework. However, constraints (3.1) make the problem very hard and solutions are not even guaranteed to be tractable. In addition, the system (under some control policies) may not be ergodic. Even if the system is stationary and ergodic, finding a closed form expression of the stationary distribution function of the queue evolution is not easy. This is due to the interaction between the different users' queues through the interference. In this Chapter, we tackle the above problem as follows. We first show that the queues of the users in the heavy traffic regime can be modeled as a reflected multidimensional SDE . Then, taking advantage of the specific structure of the reflection matrix, we propose a control policy that decouples the multidimensional SDE into several parallel SDEs and ensures that an invariant measure for each of these SDEs exists. Using results from probability theory, we can obtain a closed form expression of the stationary distribution function of the dynamics of each SDE which allows finding a relation between the reserve power allocated and the overflow probability in (3.1). Notice that the value of the reserve power allocated by our algorithm compared to the equilibrium power is very small (as we will show later in this Chapter). In other words, the sub optimality gap between our reserve power approach and other optimal control approaches (e.g. using HJB equations) is small in many scenarios.

Regarding the problem of simultaneous transmissions over interfering channels, substantial work has been done in power allocation so that the SINR at each receiver is above a specified threshold. In [20] such an algorithm, which is totally distributed, has been proposed and in subsequent years modifications and extensions have been made; for an extensive survey of power control algorithms with target SINR refer to [18] and references therein. This approach however is not suited for the traffic nature of data and video streaming applications, as they do not adapt to the traffic, queue states and/or the specific requirements of the application requested. In this direction, in [21] a scheduler based on H-infinity control was proposed in order to regulate the buffers of small cell base stations around a target length. In [22], the problem of power control for Variable Bit-Rate (VBR) video streaming over a cellular network is concerned, with the assumption that the videos requested are stored at the base stations (therefore stochastic traffic dynamics are not taken into account). The authors investigate the problem of throughput maximization under overflow and underflow constraints at the receivers' playout buffers and propose an optimal centralized and a near optimal decentralized algorithm under some feasibility assumptions for the SINR .

The authors in [23] consider dynamic scheduling (inside the cell) and power allocation (for intercell interference mitigation) so that a function of the queue lengths that corresponds to the average delay in the system is minimized. The problem is formulated as a Markov Decision Process (MDP), and an online learning algorithm is used for the solution. The proposed algorithm is semi-decentralized in the sense that a central controller sets the transmission power levels but the scheduling decision is taken at each BS . For a survey of the use of such tools in resource allocation problems see [24] and references therein. However, in these problems the objective is to optimize a single objective function subject to constraints on the expected values of the queues, which are weaker than the overflow probabilities we consider here. In addition, these techniques require the solution of the Bellman equation which in general can be solved off-line only numerically at a high computational cost, and learning algorithms for on-line implementation may converge slowly.

Another line of work regarding resource allocation in wireless networks is done by using Lyapunov drift techniques (see [25], [24] and references therein). These works address the problem of minimizing a cost function for the network while keeping the queues stable. However, in our work we are interested in satisfying individual QoS constraints for each user, in a form which is much stronger than requiring just stability of the queues. Finally, another approach is to convert the delay constraints into equivalent rate constraints, however queue state information is not taken into account and it works well for relatively large delays [24].

The approach followed in this chapter is based on the heavy traffic asymptotic modelling of a network. Initially used for the analysis of queues and queueing networks, it consists of examining the system's behaviour as the arrival rates become almost as big as the service rates. It turns out that the models in this asymptotic case become more tractable and their study can reveal useful information for the system's behaviour even when this condition does not hold. Also, due to its tractability, the heavy traffic asymptotic regime can be used to find analytically a control policy in the network (e.g. routing policy, transmission scheduling, service rate adjustment etc.); this policy then can be applied in situations where the network is not necessarily heavily loaded, with some proper modifications (see [26] and references therein).

In the context of wireless communications, heavy traffic models have been used to analyze the performance of the MaxWeight and Exponential scheduling algorithm in [27] and [28] respectively. Moreover, authors in [29] have studied the performance of a throughput-optimal rate allocation in a two-user MIMO system (with a common channel for the users) when the incoming traffic is nearly equal to the rate of each user. This was generalized in [30] for multiple antennas at the transmitter, each serving one user. In both cases the behaviour of the queues turns out to be a multidimensional Brownian motion constrained in the positive orthant. Also, in [31], it is proved that for the latter case and time-varying channels the policy of assigning rates at the boundary of the capacity region is asymptotically optimal in the sense that it minimizes a weighted sum of packets in the queue in heavy traffic; this quantity is proven to be a reflecting Brownian motion with regime switching.

The first application of the heavy traffic approach to the power control problem was made by [32], where it was used to derive an optimal power control in the setting of a single base station serving many users via time-varying but orthogonal channels. In that work, each user is preassigned one channel and the authors assumed total power constraints and that power can be reallocated from one channel to another. The control policy was specified numerically. Simulation results of this policy can be found in [33]. Also, in [34] an optimal power control was derived for the point-to-point link over a fading channel. It was shown that the delay-optimal policy is a simple single-threshold one and simulation results show that the resulting cost is very close to the one obtained by solving the original control problem. In the latter three works, heavy traffic condition is imposed by preallocating a suitable amount of power according to the channel state and then allocating a (much smaller) amount or reserve power according to channel states and queue lengths. Indeed, without this additional reserve power allocation the delay becomes unbounded [34], [35].

The main contributions of the work presented in this Chapter are: (*i*) Derivation of a heavy traffic asymptotic model for this system of K interfering wireless links for transmitters equipped with single and multiple antennas, (*ii*) Derivation of a closed-form power (and precoding in the case of multiple-antenna transmitters) control policy under perfect channel and queue state information so that the objectives (3.1) are met and (*iii*) Modification of the obtained algorithms in less demanding information patterns. We derive the analytical model of the system under heavy traffic conditions in Sections 3.2 and 3.3, extending the results of [32] and [34]. The main issue in our case is that the transmitting power of each base station affects the rates of all other wireless links, therefore the queue length processes are coupled. Then, once we obtain the SDE that models the evolution of queues, we apply a special form of linear control to the reserve power to actually decouple the evolution of queues and meet the QoS requirement in Section 3.4. In that Section we also discuss an implementation of the algorithm in more decentralized settings, where channel states are not known and the case where queue length information becomes available with delays. All the analysis up to that point is done for continuous time and an asymptotic system as a parameter n goes to infinity; in Section 3.5 we examine the effect of operating in timeslots and how the parameter n can be chosen in a practical system to actually implement the power control policies. The performance of the control algorithms is then illustrated via simulation of a simple system in Section 3.6. In this section we also illustrate via simulations the effect of delayed queue state information. Finally, Section 3.7 concludes the Chapter.

3.2 System Model and Heavy Traffic Conditions

We consider a system of K transmitters each with L antennas serving one receiver and using bandwidth W. For notational simplicity, we index the transmitters and receivers so that transmitter k serves user k. Let then $g_{ij}(t)$ denote the power gain of the channel between transmitter j and user i at time t and $\mathbf{h}_{ij}(t)$ the corresponding channel vector.

Each of these channel gains is assumed to evolve independently of the others as an ergodic finite state Markov chain (for the case of single antenna this model is widely used for fading wireless channels [92]). Under this assumption, the matrix $\mathbf{H}(t) = [\mathbf{h}_{ij}(t)]$ of all channel gains at time t will also evolve as an ergodic finite state Markov chain with, say, M_H possible states, indexed as $S_H = \{1, ..., M_H\}$. We shall denote the event that the channel gains are in the m-th state as $\mathbf{H}(t) = \mathbf{H}_m$.

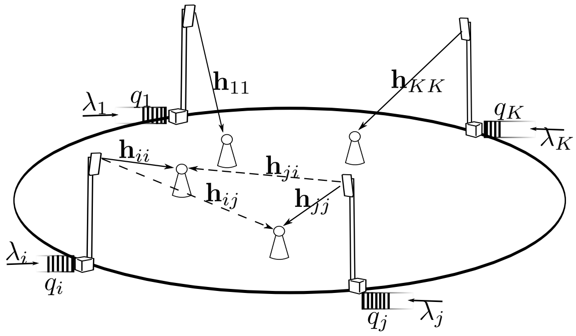

Figure 3.1: Illustration of the system model of the interference channel with queues at the transmitters

The corresponding ergodic probability distribution for each state will then be denoted as π_m, and let $\mathbb{E}_\pi\{\}$ denote the expectation over this probability distribution. In our setting no transmitter cooperation, e.g. in the form of joint transmission or space-time coding, is assumed. Therefore, at receiver k, the received signals from a transmitter other than k are regarded as interference. In addition, each transmitter will perform a power/precoding control at each time slot. Notice that time sharing and scheduling are not considered in this Chapter since they require the existence of a central entity that assigns the users to time slots. Existence of such entity is not possible in our context since we are examining a system with distinct transmitters, each of which communicating only with its own receiver. The interference is treated as Gaussian noise, thus when transmitter k uses precoder $\mathbf{v}_k$, the rate $r_k(\mathbf{v}_1(t), ..., \mathbf{v}_K(t), \mathbf{G}(t))$ over the corresponding link will be assumed as the Shannon rate, that is [93]

$$r_k(\mathbf{v}_1, ..., \mathbf{v}_K, \mathbf{H}) = W \log_2 \left(1 + \frac{|\mathbf{v}_k^H(t)\mathbf{h}_{kk}|^2}{\sigma^2 + \sum_{i \neq k} |\mathbf{v}_i^H(t)\mathbf{h}_{ik}|^2}\right). \tag{3.2}$$

In the above, W is the bandwidth and σ^2 is the noise variance. The power of transmitter k is given by $p_k(t) = \mathbf{v}_k^H(t)\mathbf{v}_k(t)$. In the case of transmitter having single antenna, each channel can be characterized only by its power gain, $g_{ki}(t)$ and the transmitted power is controlled directly. Thus, the received power in the channel from transmitter k to receiver i is $p_k(t)g_{ki}(t)$. The rate $r_k(\mathbf{p}(t), \mathbf{G}(t))$ is given by $r_k(\mathbf{p}(t), \mathbf{G}(t)) = Wlog_2\left(1 + \frac{p_k(t)g_{kk}(t)}{\sigma^2 + \sum_{i \neq k} p_i(t)g_{ik}(t)}\right)$. For each queue we suppose that the instantaneous arrivals $a_k(t)$ at time t are i.i.d., with mean λ_k and (finite) variance $\sigma^2_{a,k}$, and are independent of the arrivals at the other queues and the channel process.

In this work the heavy traffic asymptotic modelling will be used. Informally this means that the average transmission rate at each transmitter will be almost equal to the mean rate of the incoming traffic. Formally, as it is fairly standard in the relevant literature [26], a sequence of systems parametrized by n is created and the system in the limit as $n \to \infty$ is taken and examined (with time and state variables scaled appropriately). More specifically, the interpretation of the parameter n is such that at any time interval Δt there are $O(n\Delta t)$ arrivals, thus n can be seen as the order of magnitude of the arrivals and taking the limit as it goes to infinity implies that in the heavy traffic situations there are too many arrivals in the transmitters. Let $a_k^{(n)}(t)$ denote the arrival process at transmitter k at the n-th system and $\zeta_k^{(n)}(t)$ the corresponding inter-arrival times. Then, for every transmitter k we make the following assumptions [34]:

Assumption 3.2.1. *The inter-arrival intervals satisfy the following:*

1. $|\zeta_k^{(n)}(l)|^2$ *are uniformly integrable.*
2. *There exist constants* $\bar{\zeta}_k^{(n)}$, $\bar{\zeta}_k$ *and* σ_k *such that*
   $\mathbb{E}\left\{\zeta_k^{(n)}(l)\right\} = \bar{\zeta}_k^{(n)} \to \bar{\zeta}_k$ *and* $\lim_{n\to\infty} \mathbb{E}\left\{\left(1 - \frac{\zeta_k^{(n)}(l)}{\bar{\zeta}_k^{(n)}}\right)^2\right\} = \sigma_k$
3. *The inter-arrival processes are independent of the channel processes.*

The above assumption is roughly equivalent to saying that, for our case, $a_k^{(n)}(t) \to a_k(t)$, where $a_k^{(n)}(t)$ have mean rate $\lambda_k^{(n)} \to \lambda_k$ and $\sigma_{a,k}^{(n)} \to \sigma_{a,k}$. We would like to stress that λ_k and $\sigma_{a,k}$ are finite for every user k. Also, denote $\mathbf{v}_k^{(n)}(t)$ the precoding vector at time t for transmitter k of the n-th system. Define $\mathbf{v}^{(n)}(t)$ the column vector containing these precoding vectors.

A difference in our case with respect to conventional heavy traffic models in wireline networks is that the channels are changing, therefore the time scaling must be done based on the rate on which the channels change rather than the arrival rates. Following [32] and [34], we also parametrize the number of channel changes with the integer n and assume that the channels change also fast but at a slower rate than the incoming traffic. Thus, at a time interval Δt there will be $O(n^\nu \Delta t)$ channel changes, for a $0 < \nu < 1$. Now, let $q_k(t)$ denote the queue length of transmitter k at time t,and $x_k^{(n)}(t)$ the scaled version as follows:

$$x_k^{(n)}(t) = \frac{1}{n^{\frac{\nu}{2}}} q_k(n^\nu t). \tag{3.3}$$

The heavy traffic condition regarding the arrival and departure rates will be for every k [34]:

$$\lim_{n\to\infty} \left(\lambda_k^{(n)} - \mathbb{E}_\pi\left\{r_k(\mathbf{v}^{(n)}(t))\right\}\right) n^{\frac{\nu}{2}} = \theta_k < 0, \forall k \in \{1, ..., K\}, \tag{3.4}$$

The meaning of the constant θ_k in the equation is that this limit is finite. More specifically, the equation implies that the service rates are bigger than the respective arrival rates, so that the system is stable, but the gap between them closes as $O\big(1/n^{\frac{\nu}{2}}\big)$. As $n \to \infty$, the gap between the arrival and service rates becomes very small, thus activating the heavy traffic condition. In order for (3.4) to hold, the precoding vectors appearing inside the limit have to be of the form

$$\mathbf{v}_k^{(n)}(t) = \bar{\mathbf{v}}_k(\mathbf{H}^{(n)}(t)) + \frac{1}{n^{\frac{\nu}{2}}}\mathbf{v}'_k(\mathbf{x}^{(n)}(t), \mathbf{H}^{(n)}(t)). \tag{3.5}$$

In the above expression $\bar{v}_k(\mathbf{H}^{(n)}(t))$ is such that

$$\lambda_k = \mathbb{E}_\pi \left\{ r_k(\bar{\mathbf{v}}(\mathbf{H}^{(n)}(t))) \right\}, \forall k \in \{1, ..., K\}. \tag{3.6}$$

From the two equations just presented, it follows that the resource allocation policy consists of two parts: One, denoted $\bar{\mathbf{v}}_a$, the column vector of all the precoding vectors of the first part of (3.5), based only on the channel states so that the average rate equals the average rate of the incoming traffic (therefore in a sense of equilibrium) and another one, modelled by $\mathbf{u} = [\mathbf{v}'_1(t)^T, ..., \mathbf{v}'_K(t)^T]^T$ in (3.5), based on the queue lengths and probably the channel states. Throughout this work we assume that the arrival rates belong to the rate region achieved by precoding/power control only, and the main objective is finding a small reserve power such that the QoS constraints (3.1) are satisfied. In other words, when a power control policy following this rule is implemented, the equilibrium power that corresponds to the realization of the channels at this time is computed and a much smaller additional reserve power so that the total rate is very close to the arrival rate is allocated according to a rule based mainly on the queue lengths. Taking also into account that $n \to \infty$, (3.5) implies that at each time t the actual precoder used by each transmitter is varying slightly around the equilibrium according to the queue lengths so that the probabilities that the queue length exceeds a threshold are indeed the desired ones. In the topology of interference channels we are examining here, the limiting factor is interference rather than power constraints so we do not consider such constraints. However, note that if there are power constraints such that (3.6) cannot hold for some users, these queues will be unstable. On the other hand, since the reserve power is very small, a reasonable power restriction would be to add a fraction of the equilibrium power to the peak power for each user.

Examining (3.6) further, we can see that there are many possible approaches for the equilibrium power allocation. For example, one may allocate $\bar{\mathbf{v}}(t)$ so that for any possible state of the channels the rate at each link k will be equal to λ_k. Another approach is to solve the problem of minimizing the expectation over the channel states of total power used with the constraint that (3.6) is satisfied. As a final remark, unlike the networks considered in the classic heavy traffic literature where the service rates are fixed, in our case the service rates are controlled, depending on the power allocation. This means that the system is actually forced to operate in the heavy traffic regime, with its precoder being as described in (3.5) and (3.6) for some large enough scaling parameter n.

Finally, some comments with respect to notation are in order. The precoding vectors are all stacked in the column vector $\mathbf{v}(t)$ with KL elements. Therefore, element i of this vectors corresponds to transmitter $k'(i) = \lceil i/L \rceil$ and its antenna indexed by $l'(i) = i-(k'(i)-1)L$. The same holds for the equilibrium and reserve precoding vectors.

3.3 Convergence to an SDE

In this section the actual model of the limit system (as $n \to \infty$) in heavy traffic will be obtained as a controlled SDE . The derivations make use of the weak convergence methods applied in [32] and [34]. We consider the case when the channel coefficients are real numbers. This is done mainly for mathematical convenience. A way to use the results and methods of this subsection for this more realistic model (with complex channel coefficients) is to examine the power control problem separately for the sine and cosine parts of the signal; then the problem is reduced to two subproblems with real channel coefficients each, and each subproblem is solved with the same way presented in the Chapter. Our main result follows :

Theorem 3.3.1. *Consider K interfering links with L antennas at each transmitter. As $n \to \infty$ the vector-valued process of the scaled queue lengths of (3.3) is given as,*

$$\mathbf{x}(t) = \mathbf{x}(0) - \int_0^t \mathbf{f}(\mathbf{u}(s))ds + \mathbf{\Sigma}\mathbf{w}(t) + \mathbf{z}(t). \tag{3.7}$$

In the above, $\mathbf{w}(t)$ is a vector of K independent standard Wiener processes, $\mathbf{f}$ is the vector of the functions

$$f_k(\mathbf{u}(t)) = \sum_{m=1}^{M_H} \pi_m \sum_{j=1}^{LK} a_{k,j}\left(\mathbf{H}_m\right) u_j(t) \tag{3.8}$$

where $a_{i,j}(\mathbf{H}_m) = \frac{\partial r_i(\bar{\mathbf{v}}_a(\mathbf{H}_m))}{\partial v_j}$. The matrix $\mathbf{\Sigma} = [\sigma_{ij}]$ satisfies

$$\mathbf{\Sigma}\mathbf{\Sigma}^T = \mathbf{\Sigma}_a\mathbf{\Sigma}_a^T + \mathbf{\Sigma}_d\mathbf{\Sigma}_d^T \tag{3.9}$$

with $\mathbf{\Sigma}_a = diag(\sigma_{a,k})$ *while the elements of the covariance matrix* $\mathbf{\Sigma}_d\mathbf{\Sigma}_d^T = [s_{ij}]$ *are given as*

$$s_{ij} = 2\mathbb{E}\left\{\int_0^{+\infty} \hat{r}_i(0)\hat{r}_j(t)dt\right\}, \tag{3.10}$$

where $\hat{r}_k(t) = (r_k(\bar{\mathbf{v}}(\mathbf{H}(t))) - \lambda_k)$. *Finally, the elements of* $\mathbf{z}(t)$ *are given as*

$$z_k(t) = \left[-\min_{s\leq t}\left\{x_k(0) - \int_0^t f_k(\mathbf{u}(s))ds + \sum_{j=1}^{K}\sigma_{kj}w_j(t)\right\}\right]^+. \tag{3.11}$$

Proof. Please refer to the Appendix in 3.8 for the proof. □

It is interesting to note that, despite most of the cases discussed in the literature (e.g. [32], [26]), in our case the queue lengths are not directly coupled due to the reflection. Generally, the actual reflection term consists of the above process multiplied from the left by a matrix $\mathbf{R}$, denoted in literature as the reflection matrix. The diagonal elements of the reflection matrix are ones, while the off-diagonal elements represent data being routed for transmission to queues that are empty (in other words, if a queue is empty, then it serves some of the data from other non-empty queues). In our case no sort of such cooperation between the transmitters is assumed, so the reflection matrix is the identity. This property will play an important role in selecting the reserve control policy in Section 3.4.

From now on we will consider the case with single antenna transmitters in order to simplify the notation. This case follows from Theorem 2 putting $L = 1$, and we have only power control. In this case the channel gains are scalars $g_{ij}(t)$ with $\mathbf{G}(t)$ the corresponding channel gain matrix, having M_G possible states. Note that this model can also accommodate the case where the direction of the precoder in the complex space is fixed at the direction of the equilibrium vector and the reserve part only changes its magnitude.

An interesting thing to note is that we can further simplify the model by considering that the reserve power depends only on the queue lengths. This is reasonable because we can argue that the goal of the reserve power is to regulate the queue lengths while the equilibrium allocation takes care of the channels. In this case we have the following result:

Corollary 3.3.1. *Consider a reserve control policy that does not depend on the channel states. Then the asymptotic model reduces to*

$$d\mathbf{x}(t) = \mathbf{B}\mathbf{u}(\mathbf{x}(t))dt + \mathbf{\Sigma}d\mathbf{w}(t) + d\mathbf{z}(t) \tag{3.12}$$

Proof. In differential form (with the differentials being in the sense of Ito calculus) we have

$$d\mathbf{x}(t) = -\mathbf{f}(\mathbf{u}(t))dt + \mathbf{\Sigma} d\mathbf{w}(t) + d\mathbf{z}(t) \tag{3.13}$$

with an initial condition $\mathbf{x}(0) = \mathbf{x}_0$.

If $\mathbf{u}(t)$ does not depend on the channel state, we can change the order of the sums in (3.8), so

$$f_i(\mathbf{u}(t)) = \sum_{m=1}^{M_G} \pi_m \sum_{j=1}^{K} a_{i,j}(\mathbf{G}_m) u_j(\mathbf{x}(t)) = \sum_{j=1}^{K} u_j(\mathbf{x}(t)) \sum_{m=1}^{M_G} \pi_m a_{i,j}(\mathbf{G}_m)$$

thus, defining

$$b_{ij} = -\sum_{m=1}^{M_G} a_{i,j}(\mathbf{G}_m)\pi_m, \tag{3.14}$$

we get $f_i(\mathbf{u}(t)) = -\sum_{j=1}^{K} b_{ij} u_j(\mathbf{x}(t))$. Defining the matrix $\mathbf{B} = [b_{ij}]$, we can write this relation in vector form as $\mathbf{f}(\mathbf{u}(t)) = \mathbf{B}\mathbf{u}(\mathbf{x}(t))$. This implies that (3.13) takes the form (3.12), essentially completing the proof. Moreover, the elements of the matrix $\mathbf{B}$, from (3.14) are

$$b_{ii} = -\sum_{m=1}^{M_G} \pi_m W \ln(2) \frac{g_{ii}(\mathbf{G}_m)}{\sum_{k=1}^{K} g_{ki}(\mathbf{G}_m)\bar{p}_k(\mathbf{G}_m) + \sigma^2}$$

$$b_{ij} = \sum_{m=1}^{M_G} \pi_m W \ln(2) \frac{g_{ii}(\mathbf{G}_m)}{\sigma^2 + \sum_{k=1}^{K} g_{ki}(\mathbf{G}_m)\bar{p}_k(\mathbf{G}_m)} \times \frac{g_{ji}(\mathbf{G}_m)\bar{p}_i(\mathbf{G}_m)}{\left(\sigma^2 + \sum_{k\neq i} g_{ki}(\mathbf{G}_m)\bar{p}_k(\mathbf{G}_m)\right)}, i \neq j$$

□

Note that in this case, the total transmission power takes still into account both the CSI, through the equilibrium part, and the queue states, through the reserve part. For the multiple antenna case the model will be similar, just with B being a K-by-KL matrix.

As a final remark for this Section, let us point out that the whole procedure of rescaling time and queue lengths, taking the limit as this scaling factor goes to infinity and using central limit theorems implies that the Stochastic Differential Equation (3.13) is an averaged model of the system over the random environment and traffic, where statistics up to second order are used.

3.4 A Control Policy

3.4.1 Defining equilibrium and reserve control policies with perfect information

As can be seen by (3.5) the resource allocation policy consists of two parts: determining the equilibrium power allocation according to the state of the channels and then determining the reserve power allocation according to the channels and queue lengths in general. In this work, we will set the equilibrium power allocation such that the rates at each possible state of the channels are equal to the mean rates of the incoming traffic. This is equivalent to the problem of maintaining a constant $\mathsf{SINR}\bar{\gamma}_k$ for each user k for each channel state such that $\lambda_k = W \log_2 (1 + \bar{\gamma}_k)$. Therefore, the equilibrium power allocation policy is obtained by solving the following system of linear equations for each $\mathbf{G}_m$ and $\forall k \in \{1, ..., K\}$:

$$\frac{1}{\bar{\gamma}_k} g_{kk}(\mathbf{G}_m)\bar{p}_k(\mathbf{G}_m) - \sum_{i \neq k} g_{ik}(\mathbf{G}_m)\bar{p}_i(\mathbf{G}_m) = \sigma^2 \tag{3.15}$$

Here we assume that the channels and incoming traffic characteristics are such that (3.15) are feasible. If not, then we may still be able to find a suitable equilibrium power allocation; however the general problem in this case is very difficult. Also, methods using time-sharing and scheduling would need the existence of a central authority to schedule transmissions, which is not assumed in this Chapter. In addition other methods would not be amendable to distributed implementation, as described in the next subsection. In the case of multiple antennas at the transmitters, the problem of finding an equilibrium beamformer so that the corresponding rate is λ_k for each link can be solved by the algorithm presented in [19].

The reserve power allocation will depend only on the queue lengths, thus the queue dynamics are governed by a stochastic differential equation of the form (3.12). Note that because the equilibrium allocation given by (3.15) is such that the corresponding rates are the same for each channel state, there is no randomness in the equilibrium rates therefore there must be $\mathbf{\Sigma}_d = \mathbf{0}$. In order to find this reserve power allocation policy we will initially work with the asymptotic model (as $n \to \infty$) and impose no constraints on $u_k(\mathbf{x}(t))$. Having said that, we have the following;

Proposition 3.4.1. *With the equilibrium allocation given as a solution to (3.15), the overflow requirements for the asymptotic system can be satisfied by* *the following policy:*

$$\mathbf{u}(\mathbf{x}(t)) = \mathbf{B}^{-1}\mathbf{C}\mathbf{x}(t), \tag{3.16}$$

with $\mathbf{C} = \operatorname{diag}(-|c_k|)$ *and* $|c_k| = \frac{1}{2}\left(\frac{\sigma_{a,k}}{x_k^{thr}} erfc^{-1}(\delta_k)\right)^2$. *In the above,* x_k^{thr} *is the corresponding threshold in the asymptotic regime.*

Proof. Let us first start by establishing the probabilistic framework of the stochastic differential equation in (3.12). Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ be a complete probability space satisfying the usual hypotheses, i.e., $\mathcal{F}_o$ contains all the $\mathcal{P}$-null sets of $\mathcal{F}$ and $\{\mathcal{F}_t\}_{t\geq 0}$ is a right continuous filtration of σ-algebras. The Wiener Process $\mathbf{w}(t) = (w_1(t), \ldots, w_K(t))^T_{t\geq 0}$ is $\{\mathcal{F}_t\}_{t\geq 0}$-adapted with stationary and independent increments. The Wiener process is also independent of the initial state $\mathbf{x}_0$ which is an $\mathcal{F}_o$-measurable random variable with finite second moment. The reflection process $z(t) = (z_1(t) \ldots z_K(t))^T_{t\geq 0}$ is a continuous non-decreasing $\{\mathcal{F}_t\}_{t\geq 0}$-adapted $\mathbb{R}_+^K$ valued process and each $z_k(t)$ increase only when $x_k(t) = 0$. We define our control policy in the class of Markov feedback control (i.e. the control depends on $\mathbf{x}(t)$). It is well known that the existence of Markov control is related to the existence of solution for the corresponding SDE. First we will examine the effects of the proposed equilibrium policy given by solving (3.15). Since the corresponding rates are equal to each channel state, there is no randomness in the equilibrium rates therefore the diffusion matrix is now just $\Sigma = \Sigma_a$. Taking that into account and applying the proposed controller, the states are decoupled as

$$dx_k(t) = -|c_k|x_k(t)dt + \sigma_{a,k}dw_k(t) + dz_k(t). \tag{3.17}$$

Our control policy makes the evolution of the queues decoupled which is very useful to ensure the existence of a solution to the SDE . Since the states are decoupled and using the reflection direction in (3.11), we can show that the controlled process $\mathbf{x}(t)$ is positive recurrent. Using the main result of [94], the controlled process in (3.17) has a unique invariant probability measure which is absolutely continuous with respect to the Lebesgue measure, i.e. has a density that can be obtained using the Fokker-Planck equation as $\phi(x_k) = \sqrt{\frac{|c_k|}{\pi\sigma_{a,k}^2}} e^{-|c_k|x_k^2/\sigma_{a,k}^2}$. According to the ergodic properties of recurrent diffusion processes, we can use the above density of the invariant measure to compute the overflow of the controlled stochastic process x(t)

$$\mathbb{P}\left\{x_k(t) > x_k^{thr}\right\} = erfc\left(\frac{x_k^{thr}\sqrt{2|c_k|}}{\sigma_{a,k}}\right). \tag{3.18}$$

Replacing the overflow probability with its desired value and solving (3.18) completes the proof. $\square$

Note that the control policy regarding the reserve power is a closed form expression of the queue lengths. Given that the equilibrium power allocation is precomputed and under the assumption of complete queue and CSI knowledge this implies that at each time the control policy can be implemented in one shot instead of having an iterative algorithm (as it is done e.g. in [23]). In addition, let us point out that the above results are in the steady state, i.e. in the sense that the process is running for an infinite time horizon. Moreover, looking at (3.8) we can observe that the drift term of the Stochastic Differential Equation of the limit model is in fact an expectation over the ergodic distribution of the channel gains process, therefore the overflow probabilities calculated here are approximations - but quite accurate ones for many cases as we will see in the simulations section.

Finally, let us point out that the form of the power control policy is heuristic such that the overflow constraints (3.1) are satisfied. Restricting the reserve power to depend only on the queue lengths and be linear with respect to them simplified the problem quite substantially. In principle it is possible that another choice of this control may still lead to the desired result with respect to the individual QoS constraints while minimizing some cost function. However, the resulting optimal control problem is intractable to solve analytically, and a solution satisfying the constraints (3.1) is not even guaranteed to exist. Also, as we will see in the simulations, the reserve power is much smaller than the equilibrium power so the heuristic performs well in terms of energy efficiency. For the multiple antenna case the procedure is the same, with the only difference that in the control policy there needs to be the pseudoinverse of the corresponding matrix $\mathbf{B}$.

3.4.2 Control policy with local SINR feedback

So far, we have assumed that at each time slot there is full knowledge of the realizations of all the channels and queue lengths and the calculation of the transmission powers was done based on that knowledge. However, this assumption is unrealistic in practice. In this subsection we will examine the case where each receiver can send SINR feedback to its corresponding transmitter but other than that no information on the channels is available. Taking into account that the equilibrium power allocation is such that the corresponding rate is constant, we can use the algorithm proposed in [20] to find the equilibrium power allocation without the need for knowledge of all channel realizations. More specifically, for a system adjusting the power in discrete time, in the beginning of each time slot we can dedicate a time τ where the transmitters find the equilibrium power.

In order to do that, each transmitter k requires only the SINR feedback from its corresponding receiver and runs the following iterative process

$$p_k(i+1) = \frac{\bar{\gamma}_k}{\gamma_k(i)} p_k(i), \tag{3.19}$$

where i denotes here the iteration of the algorithm, $\bar{\gamma}_k$ the target SINR of user k and $\gamma_k(i)$ the SINR at this user after the power update of iteration i. It is shown in [18], [20] that, for a given but unknown realization of the channels, this algorithm indeed converges to the equilibrium values corresponding to these channels and moreover this convergence is very fast (the rate of convergence is exponential). This analysis implies that the only real time information truly needed to implement the proposed power control policy are the queue lengths at the beginning of each time slot (they are still required to compute the reserve power). Also, the statistics of the channels and incoming traffic processes are still needed in order to find the parameters in the SDE that models the evolution of the queue lengths.

As a result of the training period, the actual data transfer is taking place for a duration of $(T_s - \tau)$ in a time slot of duration T_s, instead of its whole duration. A way to take this into account is to adjust the bandwidth in (2) appropriately, i.e. if W' is the physical available bandwidth, the rates and all parameters in the models are calculated using bandwidth

$$W = \frac{T_s - \tau}{T_s} W'. \tag{3.20}$$

In the above, τ is assumed fixed and taken such that it is enough for the algorithm given by (3.19) to converge.

For the case of multiple antenna transmitters, each transmitter can acquire the channel realization of its respective receiver. See [19] for an algorithm requiring little information exchange between transmitters and [95] for a decentralized algorithm to achieve the desired equilibrium rates.

3.4.3 Control policy in the case of delayed queue state information

Another major issue for the practical implementation of our control policy is the requirement that each transmitter is aware of all the queue lengths instantaneously. A more realistic assumption would be that each transmitter knows its own instantaneous queue state and has access to delayed information about the queues of the others. For example this can correspond to a SCN setting where the base stations exchange information using a backhaul of limited capacity. We will assume though that each transmitter knows the statistics of the arrivals to the other transmitters. To simplify the analysis, we will assume that the delay in sharing the queue length information is the same for any pair of transmitters, i.e. at time t, transmitter i has access to the queue length of transmitter j at time $t - \tau_d$.

Let us define the observation of the queue length of transmitter i at time t as $\hat{x}_i(t) = x_i(t - \tau_d)$ and the vector of the observations of the queue states at transmitter i as $\hat{\mathbf{x}}_k(t)$. Then the following holds

Proposition 3.4.2. *In the case where the queue state information is exchanged with delay τ_d and the control of Section 3.4 is applied, the asymptotic model evolves as*

$$d\mathbf{x}(t) = \mathbf{C}\mathbf{x}(t)dt + \mathbf{B}\mathbf{D}_L\left(\tilde{\mathbf{x}}(t) - \mathbf{x}(t)\right)dt + \boldsymbol{\Sigma}d\mathbf{w}(t) + d\mathbf{z}(t). \tag{3.21}$$

The matrix $\mathbf{D}_L$ is given as the matrix $\mathbf{L} = \mathbf{B}^{-1}\mathbf{C}$ with its diagonal elements replaced by zeros, $\tilde{\mathbf{x}}(t) = x_k(t - \tau_d)$.

Proof. Denoting $\mathbf{L} = [l_{ij}]$, the control using the delayed observations is given by $u_k(t) = l_{kk}x_k(t) + \sum_{i \neq k} l_{ki}x_i(t - \tau_d)$. In a vector form, and using $\mathbf{L} = \text{diag}\{l_{kk}\} + \mathbf{D}_L$ we get $\mathbf{u}(t) = \text{diag}\{l_{kk}\}\mathbf{x}(t) + \mathbf{D}_L\tilde{\mathbf{x}}(t) = \mathbf{L}\mathbf{x}(t) + \mathbf{D}_L\left(\tilde{\mathbf{x}}(t) - \mathbf{x}(t)\right)$. Replacing in (3.12) and taking into account that $\mathbf{L} = \mathbf{B}^{-1}\mathbf{C}$ we get the stated result. $\square$

An observation that can be made is that in the case of delayed queue length information, a term proportional to the difference between the vector of observations (delayed versions) of the queues and the vector of the actual queue lengths $\mathbf{e}(t) = \tilde{\mathbf{x}}(t) - \mathbf{x}(t)$ is added to the original evolution of the queue lengths.

In order to analyze the impact of the delay, consider the case where this delay in information sharing is infinite. In this case, no information about the queue lengths is in fact shared among the transmitters so their estimations cannot be updated. Thus, the estimation vector in (3.21) will be a constant $\tilde{\mathbf{x}}$ (which can correspond for example to an initial estimation). In this case, replacing $\tilde{\mathbf{x}}(t) = \tilde{\mathbf{x}}$ in (3.21), we get that the state evolves according to the equation $d\mathbf{x}(t) = (\mathbf{B}\mathbf{D}_L\tilde{\mathbf{x}} - \mathbf{A}\mathbf{x}(t))dt + \boldsymbol{\Sigma}d\mathbf{w}(t) + d\mathbf{z}(t)$, where $\mathbf{A} = [a_{ij}] = \mathbf{C} - \mathbf{B}\mathbf{D}_L$. This case is the worst case scenario, as each transmitter gets no information at all about the evolution of the queue lengths of the others, and can be used to find the upper bound of the overflow probabilities for any estimation scheme (as the general case with a finite delay is very difficult to be analyzed). Denote $\phi(t, \mathbf{x})$ as the density of the joint probability distribution of the queue lengths at time t. Then, from the theory of SDEs , it follows the corresponding Fokker Planck equation

$$\frac{\partial}{\partial t}\phi(t,\mathbf{x}) = -\sum_{i=1}^{K} a_{ii}\phi(t,\mathbf{x}) + \sum_{i=1}^{K} \frac{\sigma_{a,i}^2}{2}\frac{\partial^2}{\partial x_i^2}\phi(t,\mathbf{x}) \tag{3.22}$$

with the appropriate initial conditions for $t = 0$ and at the reflecting barrier (i.e. at all points $\mathbf{x}$ with at least one element zero) [96]. This equation can be solved only numerically and the invariant measure as $t \to \infty$ can be taken. Then, using the marginal distributions we can obtain numerical results for the queue overflow probabilities.

A way to use this is to adjust the threshold appropriately: If the overflow probability at some transmitters are larger than desired, we can decrease the corresponding thresholds and derive the control policy for these decreased thresholds. The overflow probability then for the initial (bigger) thresholds will be smaller. This procedure can be repeated until we get acceptable overflow probabilities for the initial desired thresholds.

For the case where the delay in information sharing is not infinite, a simple heuristic approach is that each transmitter calculates the transmission power with the queue length vector being replaced with the vector of the most recent information about the queue states this transmitter has. While there is no information yet about the queue at a transmitter i, the transmitter k uses the standard deviation of the incoming traffic at i properly modified taking into account the time slot duration, i.e. $\sqrt{T_s\sigma_{a,i}^2}$, as an estimation of the queue length. This scheme will intuitively perform better than the worst case scenario (with no queue length information of other transmitters) described earlier, however it is very difficult to analyze.

3.5 Discrete Time Implementation Issues

So far we worked on continuous time models whereas in the real communications system time is slotted and power allocation decisions are taken into discrete time instances. More specifically, let the duration of each timeslot of the unscaled system be T_s; this implies that at the n-th system of the sequence the timeslot duration will be $T_s^{(n)} = T_s n^{-\nu}$. Then, in the general case where the equilibrium allocation is such that $\mathbf{\Sigma_d} \neq \mathbf{0}$, the departure process converges to a Wiener process with covariance matrix given as

$$s'_{ij} = T_s\mathbb{E}_\pi\left\{\hat{r}_i(0)\hat{r}_j(0)\right\} + 2T_s\mathbb{E}\left\{\sum_{l=1}^{+\infty}\hat{r}_i(0)\hat{r}_j(l)\right\}$$

see e.g. [32]. The expectations are taken again with respect to the distribution of the channels and $\hat{r}_j(l)$ are the discrete time versions of the corresponding quantities given by (3.33). The analysis and explanation of the above equation is the same as in continuous time.

The derivation of the control policy was concerned with an asymptotic system model. However, it has to be modified in order to operate on a real system, so the results obtained for the asymptotic case have to be converted into results for the unscaled system. This will be done using (3.3) and (3.5), where this scaling parameter n is now a "big enough" finite number. By the definition that at a time interval δt there are $O(n\delta t)$ arrivals, we can argue that in practical cases n can be the order of magnitude of the average bit rates of the incoming traffic in the system. Also, as far as the exponent ν is concerned, once n is fixed it can be obtained using the fact that during a period with duration equal to the coherence time, T_{coh}, there must be only one channel change, thus $n^{\nu}T_{coh} = 1$.

In the specific control policy presented in the previous section, the equilibrium rates are the same with the mean rates so the covariance matrix of the limit Wiener process that corresponds to the departure process will be zero regardless of the fact that the system operates in time slots. Note however that we have the assumption that the channel changes slower than the arrival process, so we can assume that the channel stays the same within a timeslot. Based on the previous analysis, at the "real" system where the queue lengths are $\mathbf{q}(l)$ at the beginning of timeslot l and the channel gain matrix is at state m, the power allocated for the duration of this timeslot will become (applying time slotting and unscaling in the results of Section 3.4)

$$\mathbf{p}(l) = \bar{\mathbf{p}}(\mathbf{G}(l)) + \mathbf{B}^{-1}\mathbf{C}^{'}\mathbf{q}(l),$$

where $\mathbf{C}^{'} = \text{diag}(|c_k^{'}|)$ is obtained by Proposition 4 replacing the scaled queue length threshold with its real unscaled value, q_k^{thr}. Note also that since in (3.18) both quantities inside the probability are scaled ones, this is also the probability of the unscaled queue lengths exceeding their respective unscaled thresholds.

From section 3.4 it is implied that it is possible to find a control policy of the type discussed for any values of the queue thresholds and overflow probabilities, which intuitively should not be the case. In fact, some limitations are given by the following proposition:

Proposition 3.5.1. *Consider a time slotted system with slots of length T_s for data transmission that operates updating the power allocation at the start of each time slot as described in this Section. Then, for each transmitter k, the achievable queue length thresholds to be exceeded with probability δ_k are bounded* *as*

$$q_k^{thr} \geq \frac{\sigma_{a,k}}{2}\sqrt{T_s}erfc^{-1}\left(\delta_k\right) \tag{3.23}$$

Proof. The discrete time dynamics of the real system can be approximated from (3.17) taking into account the form of the expression for the total power as ($n_k(l)$ is the discrete time White Gaussian process with unitary variance): $q_k(l+1) = \left[\left(1 - |c'_k|T_s\right) q_k(l) + T_s\sigma_{a,k}n_k(l)\right]^+$. For the above difference equation not to diverge, there must be $|1 - |c'_k|T_s| \leq 1$, therefore $|c'_k| \leq \frac{2}{T_s}$. Replacing $|c'_k|$ using the analysis in section 3.4, we get the stated relation. □

This threshold bound is an approximation and it illustrates the effect of the discrete time operation in an actual system. The same results hold for the multiple antenna case. The results in this Section are based on heuristics and are approximate. Obtaining exact results for the initial discrete time system is, as far as we are aware of, generally an open issue in the area of analysing and designing systems via traffic approximations. However, these approximate results provide useful insights.

3.6 Simulation Results

In order to illustrate the results of the power control method presented in sections 3.4 and 3.5, let us consider a simple scenario with 3 interfering transmitter - receiver pairs, using the same spectrum with bandwidth $5MHz$. For simplicity, consider that each channel gain has only two possible values. Also, the arrivals at each transmitter were set as Poisson processes with mean rates 1, 1.5 and 2 Mbps. The overflow thresholds are $500, 750, 1000$ bits at each transmitter respectively and the overflow probability is 0.01 for all transmitters. The coherence time of the channels is set to $20ms$, corresponding to slow fading channels like in indoor and low mobility environments. The time slot duration is $2ms$, motivated by the shortest scheduling interval in High Speed Downlink Packet Access (HSDPA) , thus the channel stays the same for 10 consecutive power configurations. The noise variance was set to 0.01 at each receiver.

Based on the above settings and the analysis in Section 3.5, for the simulated system a reasonable value of n can be $n = 10^6$, the order of magnitude of the incoming traffic at all users, and ν can be such that $n^\nu T_{coh} = 1$, thus $\nu = 0.283$.

The maximum equilibrium power is $83mW$, while all the equilibrium powers are in the order of magnitude of mW. The expected values of the equilibrium powers over the ergodic distribution of the channel gains matrix were found to be $43mW, 62mW, 81mW$ for transmitters $1, 2$ and 3 respectively. The average reserve powers used where found (by simulations) to be around $0.7 \times 10^{-2} mW, 0.7 \times 10^{-2} mW, 0.8 \times 10^{-2} mW$.

Our proposed method is compared with the policy where all powers take their equilibrium values according to the channel states, and the following heuristic power allocation strategies: One with the power at each transmitter being constant for every queue length and channel state and equal to the expectation of the equilibrium power over the ergodic distribution of the channel gain matrix plus the average reserve power presented above and one with the power being the one given adding the equilibrium allocation. In the presentation of the results, the former will be denoted as "Static power allocation" and the latter as "Channel-aware power allocation". These configurations were made to ensure that the amount of power available in the heuristic schemes is approximately the same as the power used in our proposed method, thus making a fair comparison. We will show the results concerning the queue of one transmitter, the one for Link 1, as all the results are very similar.

The performance of the aforementioned power allocation policies for each queue are shown in Figure 3.2.

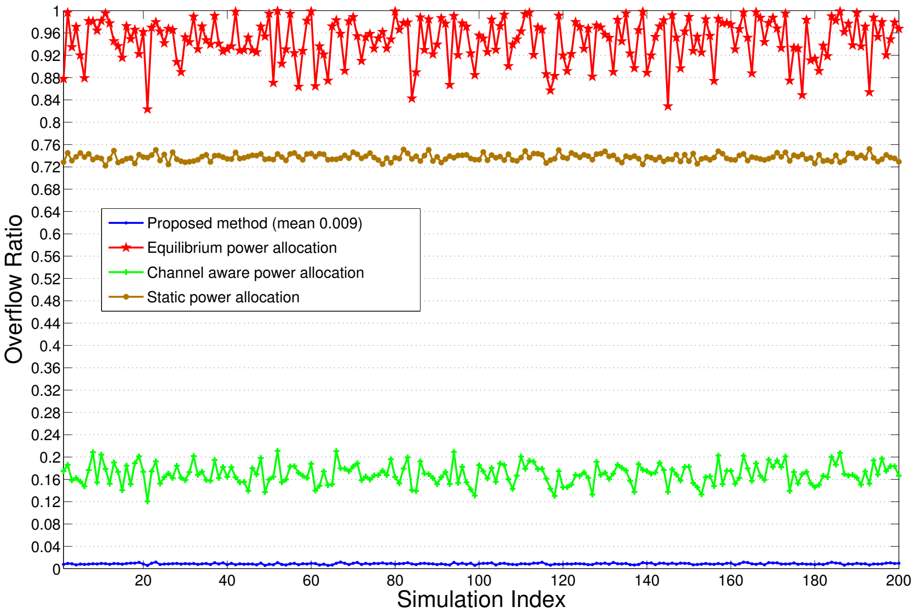

Figure 3.2: Overflow ratios for the queue at link 1 in the three links setting.

The static power allocation performs rather bad as it does not take into account at all the channel states and the queue lengths. As far as the equilibrium power allocation is concerned, the bad performance is because the traffic and the queue lengths are not taken into account. Analyzing this policy further, (3.12) implies that putting the vector of the reserve powers **u** equal to zero, the queue lengths behave like reflecting Wiener processes. This is illustrated in our simulation setting in Figure 3.3.

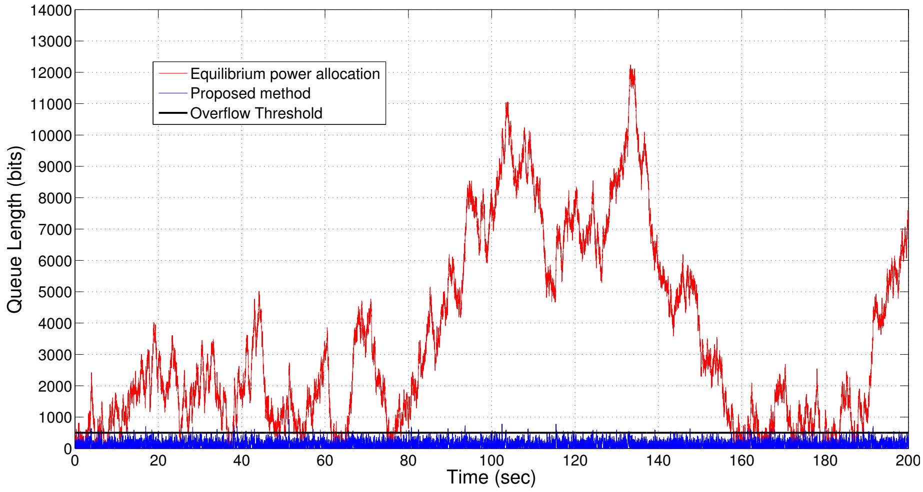

Figure 3.3: Evolution of the queue lengths at transmitter 1 in the three-links setting in one simulation run, using the equilibrium and the proposed power allocation method.

In Figure 3.2 it is clearly illustrated that assigning slightly more power in the equilibrium power for each channel state, as it is done in the channel-aware power allocation scheme defined earlier, leads to a much better performance, even if this extra power is very small. This was also expected as an increase in all the powers leads to an increase in the average rates. The equilibrium power allocation is exactly the point where the mean arrival rates equal the mean service rates thus even a small increase of the mean service rates is enough to stabilize the system. Achieving a better performance than the static power allocation case was also expected due to the transmission powers adapting to the channel conditions. Finally, we can observe that our proposed method of power allocation does even better, illustrating the additional advantage of taking the queue lengths into account when allocating the reserve power. Moreover, the overflow ratio is very close to the desired one, which implies the validity of the asymptotic model in a practical system operating under heavy load.

In Figure 3.3 we can also see the evolution of the queue length over time for a simulation run. We can observe that the queue length under the proposed power control method behaves in a much more controlled manner compared to the equilibrium power allocation (which indeed behaves like a Wiener process, thus validating the system model) and is below its respective threshold for most of the time.

Finally, Figure 3.4 depicts the power allocated using our proposed policy and the equilibrium power allocation for the transmitter of Link 1 for the first 100 instances of a simulation run. We can see that the total allocated power

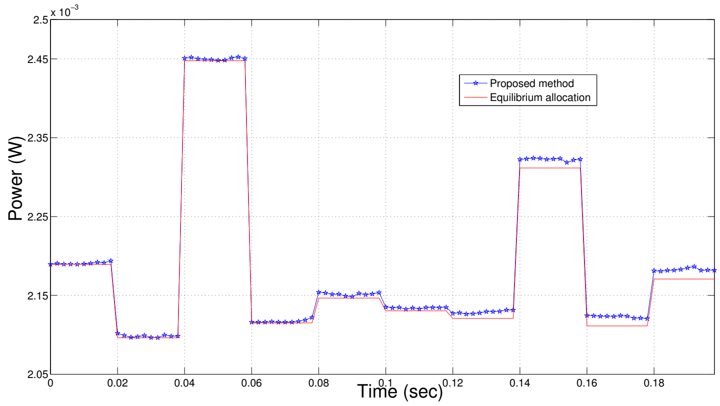

Figure 3.4: Evolution of the equilibrium and allocated power using our method for Link 1 for the first 100 timeslots of a simulation

varies slightly around the equilibrium power level in each channel state, which clearly demonstrates the effect of allocating some extra power according to the queue lengths. We can see that this variation around the equilibrium power is indeed very small, thus confirming the assumption of very small reserve power.

As a next step, a simple system with transmitters with multiple antennas is simulated in order to verify the theoretical results and illustrate the impact of multiple antennas in the operation of the system. For computational purposes, we consider a network of two links for the cases where the transmitters have one, two and three antennas. The channels from each antenna to each receiver are assumed independent and identically distributed (i.i.d.) two state Markov chains and the incoming traffic processes are again Poisson distributed with mean rates 1 and 2 Mbps for links 1 and 2 respectively. We set the respective thresholds to 500 and 1000 bits and the desired overflow probability to 0.01. For any given number of antennas, both precoding control methods (i.e. fixed direction for a given realization of channel states and direction of the reserve vector depending on the queue length) are considered, operating both for the same instances of incoming traffic and channel realizations.

A result from these simulations is that for each simulation run and a given number of antennas (that is the same traffic and channel realizations) the overflow ratios were the same for each precoding method used. Indeed, from the theoretical analysis it holds that for any precoding method presented here, the control is such that evolution of the queues follows the same equation (3.17). Moreover, Figure 3.5 shows the Cumulative Distribution Function (CDF) of the overflow probabilities over the simulation runs; that is for each curve the y axis shows the ratio of simulation runs where the overflow ratio was smaller than the corresponding point of the curve at x axis. We can see that the overflow ratio

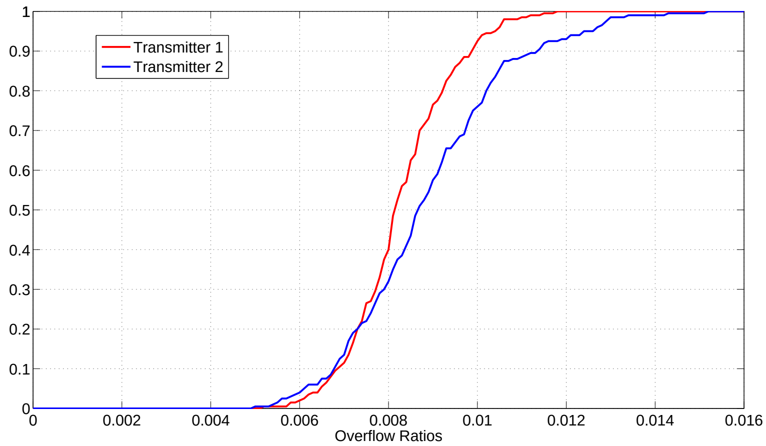

Figure 3.5: CDF over the 200 simulation runs of the overflow ratios for 2 transmitter-receiver pairs and 3 antennas per transmitter.

was indeed very close to the desired one. These results illustrate the validity of the asymptotic approach in a practical system. Regarding the evolution over time, it is similar as in the case with 3 links and one antenna (see Figure 3.3).

Fig. 3.6 depicts the average total power consumption of the system in each simulation run for each of the cases examined. In the cases of multiple antennas, the average power used was almost the same for the two precoding methods, so we present just one plot for each number of antennas. These results imply that adding more antennas at the transmitters, under the assumption that the corresponding paths are independent, increases the energy efficiency of the system for the same requirements in terms of buffer overflows.

Finally, we study through simulations the effect of delayed queue state information in our algorithm. For simplicity, all the delays in information sharing between the transmitters are set to be the same. Performance in terms of overflow ratios are given in Fig. 3.7 as the CDFs over the simulation runs (in the

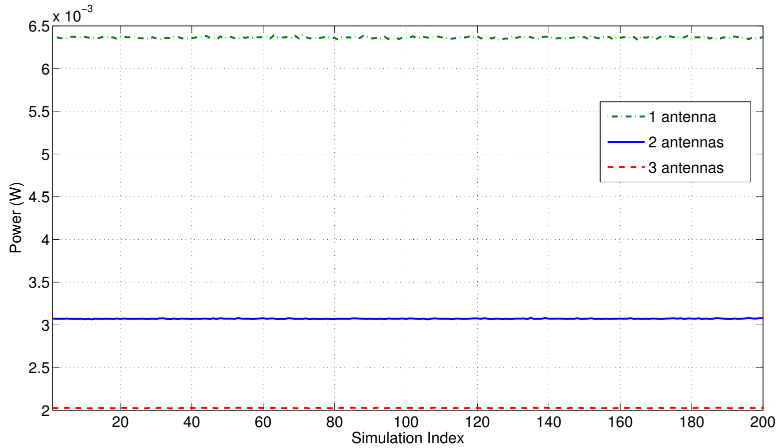

Figure 3.6: Average power usage for the 2 transmitter-receiver pair scenario and different number of antennas at the tansmitters

same sense as in Fig. 3.5).

As we can see in this figure, as the delay in information sharing increases the overflow ratios tend to be higher. However, especially for small delays, the differences are still relatively small. Also, even for relatively high delay in information sharing the overflow ratios tend to be not very far from the desired one. Thus we can argue that knowing the incoming traffic statistics at each base station, our proposed scheme seems to be quite robust in cases of delayed information sharing.

3.7 Conclusions

In this Chapter we have used the heavy traffic approximation in order to model and propose algorithms for the power allocation problem for interfering wireless links, which results in an asymptotic but tractable way to analyze the problem and derive some control strategies. The objective was to keep desired overflow probabilities at each queue assuming that the channels change according to an ergodic finite state Markov chain. The allocated power is split into two: a part that is allocated according to the channels and a much smaller part that is allocated according to the backlogs at the queues at each time, for which closed form expression as a function of the queue lengths was derived. The advantage of the algorithm is that it can be implemented in one shot at the beginning of each time slot. This work was also extended for the case when the transmitters have multiple antennas, where precoding methods were proposed following the

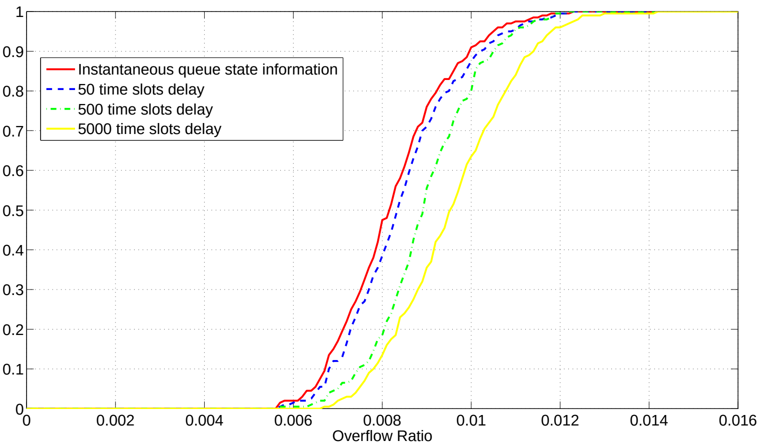

Figure 3.7: CDFs over the 200 simulation runs of the overflow ratios for the proposed algorithm implemented with delayed information or Link 1

same approach. Even though the model derived and used is an approximation, simulation results have shown that for reasonable thresholds and overflow probabilities, direct application of the policy derived from the asymptotic system can give quite accurate results. In addition, simulation results imply that the algorithm is quite robust in the case of delayed queue state information.

3.8 Appendix for Chapter 3: Proof of Theorem 3.3.1

In order to prove the convergence of the scaled queue to an SDE , we adopt an approach similar to the one used in [32, 34]. Recall that [32] deals with centralized power control in a multiuser downlink system with orthogonal transmissions among users (no interference) and single antenna system while [34] deals with power control for a point-to-point single antenna and single user wireless channel. Given the form of the precoding vectors, we can write the rates at the $n-$th system as $r_k^{(n)}(\mathbf{v}_a(t), m) = r_k(\bar{\mathbf{v}}(\mathbf{H_m}) + \frac{1}{n^{\frac{\nu}{2}}}\mathbf{u}(\mathbf{x}(t), m))$ and expand it in a Taylor series around $\bar{\mathbf{v}}(\mathbf{G_m})$ as $n \to \infty$. We will then get

$$r_k^{(n)}(t) = r_k(\bar{\mathbf{v}}(\mathbf{H}_m)) + \frac{1}{n^{\frac{\nu}{2}}} \sum_{i=1}^{LK} \frac{\partial}{\partial v_i}(r_k(\bar{\mathbf{w}}_a(\mathbf{H}_m)))u_i(\mathbf{x}(t), \mathbf{H}_m) +_O (n^{-\frac{\nu}{2}}) \tag{3.24}$$

Denoting $r_k(\bar{\mathbf{w}}(\mathbf{H}_m)) = \bar{r}_k(\mathbf{H}_m)$ (as in fact these rates depend only on the channel state), we can write the rate (3.24) as

$$r_k^{(n)}(t) = \bar{r}_k(\mathbf{H}_m) + \frac{1}{n^{\frac{\nu}{2}}} \sum_{i=1}^{LK} a_{k,i}(\mathbf{H}_m) u_i(\mathbf{x}^{(n)}(t), \mathbf{H}_m). \tag{3.25}$$

If $d_k(l)$ are the amounts of data transmitted from transmitter k at timeslot l, we have $q_k(l+1) = [q_k(l) + a_k(l) - d_k(l)]^+$. Denoting now $A_k(t)$, $D_k(t)$ the arrivals and total bits that could have been transmitted (if the queue was always full) up to time t respectively, we get

$$q_k(t) = [q_k(0) + A_k(t) - D_k(t)]^+. \tag{3.26}$$

Now let us construct the sequence of systems whose limit will be eventually the heavy traffic model of the system. Recall that $x_k^{(n)}(t) = \frac{1}{n^{\frac{\nu}{2}}} q_k(n^\nu t)$ and also let us define the centred around the mean rates versions of the total arrival and data transmission processes as :

$$\bar{A}_k^{(n)}(t) = \frac{1}{n^{\frac{\nu}{2}}} \int_0^{n^\nu t} (a_k^{(n)}(s) - \lambda_k) ds \tag{3.27}$$

and

$$\bar{D}_k^{(n)}(t) = \frac{1}{n^{\frac{\nu}{2}}} \int_0^{n^\nu t} \left(\bar{r}_k(\mathbf{H}^{(n)}(s)) - \lambda_k \right) ds \tag{3.28}$$

respectively. Also define the scaled amount of data transmitted due to only the reserve allocation up to time t (assuming always full queue) as

$$F_k^{(n)}(t) = \frac{1}{n^\nu} \int_0^{n^\nu t} \sum_{m=1}^{M_H} I_{\{\mathbf{H}^{(n)}(s)=\mathbf{H}_m\}} \sum_{i=1}^{LK} a_{k,i}(\mathbf{H}_m) u_i(\mathbf{x}(s), \mathbf{H}_m)) ds. \tag{3.29}$$

In this case, we can rewrite (3.26) as

$$x_k^{(n)}(t) = x_k^{(n)}(0) - F_k^{(n)}(t) - \bar{D}_k^{(n)}(t) + \bar{A}_k^{(n)}(t) + z_k^{(n)}(t). \tag{3.30}$$

In the above, $z_k^{(n)}(t)$ are processes to satisfy the physical constraint that each of the scaled queue lengths $x_k^{(n)}(t)$ is nonnegative.

We will now, similar to [20, 22], examine the convergence as $n \to \infty$ of each one of the terms in the above equation separately.

3.8.1 Arrival Process

Given that $a_k^{(n)}(s)$ are i.i.d. in time and that $a_k^{(n)}(s) \to a_k(s)$ where $a_k(s)$ has finite mean λ_k and variance $\sigma_{a,k}^2$, from an extension of the Functional Central

Limit Theorem [32] it holds that as $n \to \infty$, $\bar{A}_k^{(n)}(t)$ converges weakly to a Wiener process with zero drift and variance $\sigma_{a,k}^2$:

$$\bar{A}_k^{(n)}(t) \xrightarrow{w} \sigma_{a,k} w_{a,k}(t). \tag{3.31}$$

In the above, $w_{a,k}(t)$ denotes the standard Wiener process.

The above convergence happens for every k and recalling that incoming traffic flows are independent, the vector containing these arrival processes as elements converges weakly as

$$\mathbf{a}^{(n)}(t) \xrightarrow{w} \mathbf{\Sigma}_a \mathbf{w}_a(t) \tag{3.32}$$

where $\mathbf{\Sigma}_a = \operatorname{diag}(\sigma_{a,k})$ and $\mathbf{w}_a(t)$ is a vector of independent standard Wiener processes.

3.8.2 Service Process

In order to find the limit of the departure process, we will follow a method similar as before. More specifically, we can write (3.28) as

$$\bar{D}_k^{(n)}(t) = \frac{1}{n^{\frac{\nu}{2}}} \int_0^{n^\nu t} \sum_{m=1}^{M_H} I_{\{\mathbf{H}(s)=m\}} \bar{r}_k(m) ds - \lambda_k n^{\frac{\nu}{2}} t.$$

We define now $\tilde{r}_k(t) = \sum_{m=1}^{M_H} I_{\{\mathbf{H}(t)=\mathbf{G}_m\}} \bar{r}_k(\mathbf{H}_m)$; then by the definition of the indicator function the vector $\tilde{\mathbf{r}}(t)$ is a finite state Markov chain evolving as the channels with values $\bar{\mathbf{r}}(\mathbf{H}_m)$ when the channel processes are at state m. Denote now also

$$\hat{\mathbf{r}}(t) = \tilde{\mathbf{r}}(t) - \lambda. \tag{3.33}$$

Taking into account the equilibrium power allocation, we have $\lambda_k = E\{\hat{r}_k(t)\}$ and following [32] we have that the above converges weakly to a K-dimentional Wiener process with covariance matrix $\mathbf{S}_d = [s_{ij}]$ such that

$$s_{ij} = 2\mathbb{E}\left\{\int_0^{+\infty} \hat{r}_i(0)\hat{r}_j(t)dt\right\}.$$

By comparison, this process is exactly the multidimentional process that has $\bar{D}_k^{(n)}(t)$ as its k-th element. So denoting $\mathbf{\Sigma}_d = [\sigma_{ij}]$ such that $\mathbf{\Sigma}_d \mathbf{\Sigma}_d^T = \mathbf{S}_d$, we have for each k

$$\bar{D}_k^{(n)}(t) \xrightarrow{w} \sum_{j=1}^{K} \sigma_{kj} w_{d,j}(t) \tag{3.34}$$

where $w_{d,j}(t)$ independent standard Wiener processes.

It has to be noted here that, unlike the centred arrival processes, the centred transmission processes are not independent in the limit. That was expected because essentially they depend upon the same process (the one that governs the channel gain matrix). The elements of the covariance matrix of the limiting Wiener process are actually integrals of the temporal correlation between the centred processes at the queues and actually depend on the temporal correlation of the Markov chain modelling the channel gains. In the case that channel gains were i.i.d. over time, only the correlation at the same time instance (t= 0) would appear. However, as the Markov chain is ergodic, from some time onwards will have reached its invariant distribution and the correlations with the initial time would be zero. Finally, for the special case where the equilibrium allocation is such that $\bar{r}_k(m) = \lambda_k$ for every k and m, the centered departure process is just the zero process.

As far as the reserve power is concerned, in the limit as $n \to \infty$, applying the Functional Law of Large Numbers we obtain

$$F_k^{(n)}(t) \xrightarrow{w} \int_0^t \sum_{m=1}^{M_H} \pi_m \sum_{i=1}^{LK} a_{k,i}(\mathbf{H}_m) u_i(\mathbf{x}(s), \mathbf{H}_m) ds.$$

With $f_k(\mathbf{u}(s))$ given as in (3.8), this implies

$$F_k^{(n)}(t) \xrightarrow{w} \int_0^t f_k(\mathbf{u}(s)) ds. \tag{3.35}$$

For completeness, we present the expressions for the coefficients $a_{k,i}(\mathbf{H}_m)$:

$$a'_{k,i}(\mathbf{H}_m) = W \ln(2) \frac{2h_{k'(i)k}^{(l'(i))}(\mathbf{H}_m)|\bar{\mathbf{w}}_k^H(\mathbf{H}_m)\mathbf{h}_{k'(i)k}(\mathbf{H}_m)|}{\sigma^2 + \sum_{j=1}^K |\bar{\mathbf{w}}_j^H(\mathbf{H}_m)\mathbf{h}_{jk}(\mathbf{H}_m)|^2}, k'(i) = k$$

and, for $k'(i) \neq k$:

$$\begin{aligned} a'_{k,i}(\mathbf{H}_m) = -W \ln(2) & \frac{2h_{k'(i)k}^{(l'(i))}(\mathbf{H}_m)|\bar{\mathbf{w}}_k^H(\mathbf{H}_m)\mathbf{h}_{kk}(\mathbf{H}_m)|^2}{\sigma^2 + \sum_{j=1}^K |\bar{\mathbf{w}}_j^H(\mathbf{H}_m)\mathbf{h}_{jk}(\mathbf{H}_m)|^2} \\ \times & \frac{|\bar{\mathbf{w}}_{k'(i)}^H(\mathbf{H}_m)\mathbf{h}_{k'(i)k}(\mathbf{H}_m)|}{\sigma^2 + \sum_{j \neq k} |\bar{\mathbf{w}}_j^H(\mathbf{H}_m)\mathbf{h}_{jk}(\mathbf{H}_m)|^2}. \end{aligned}$$

3.8.3 Reflection Process

In order to complete the model, we have to examine the processes $z_k^{(n)}(t)$. As mentioned earlier, $z_k^{(n)}(t)$ must be such that the queue lengths remain nonnegative, thus defining what exactly happens at the time instance when the queue of transmitter k is empty (i.e. there are no data to send to its receiver). In our system setting each transmitter serves only its own receiver and the equilibrium power is assigned even if the queue is empty at the power allocation instance. Therefore, when a base station is allocated power when its queue is empty, the power is wasted and the process $z_k^{(n)}(t)$ corresponds to just the amount of data that could have been sent during this transmission; this amount has to be subtracted from the result of the other terms in (3.30):

$$z_k^{(n)}(t) = \frac{1}{n^{\nu/2}} \int_0^{n^\nu t} (r_k^{(n)}(s) - a_k^{(n)}(s)) I_{\{x_k^{(n)}(s)=0\}} ds.$$

The above implies that this process increases only when $x_k^{(n)}(t)$ hits zero. The reflection process that satisfies these requirements is [21]

$$z_k^{(n)}(t) = \max\left\{0, -\min_{s\leq t}\left\{x_k^{(n)}(0) + A_k^{(n)}(s) - D_k^{(n)}(s)\right\}\right\}. \tag{3.36}$$

Indeed, note that

$$\max\{0, -\min_{s\leq t}\{x_k^{(n)}(0) - D_k^{(n)}(t) + A_k^{(n)}(t)\}\} \geq -\left(x_k^{(n)}(0) - D_k^{(n)}(t) + A_k^{(n)}(t)\right).$$

Replacing in (3.30), we get that

$$x_k^{(n)}(t) \geq x_k^{(n)}(0) - D_k^{(n)}(t) + A_k^{(n)}(t) - \left(x_k^{(n)}(0) - D_k^{(n)}(t) + A_k^{(n)}(t)\right) = 0,$$

therefore the scaled queue lengths are kept nonnegative. We now focus on the cases where $x_k^{(n)}(t)$ tends to become negative if no reflection was present. Let t_1 the smallest time instance where $x_k^{(n)}(t^-) = 0$ such that $x_k^{(n)}(t_1) < 0$, that is when the process of scaled queue lengths hits zero with a direction to get negative. Therefore $x_k^{(n)}(0) + A_k^{(n)}(t_1) - D_k^{(n)}(t_1) < 0$. Then, for $t < t_1$, $z_k^{(n)}(t) = 0$ and for $t = t_1$ we have $z_k^{(n)}(t_1) = -(x_k^{(n)}(0)+A_k^{(n)}(t_1)-D_k^{(n)}(t_1)) > 0$, therefore the process increases the first time $x_k^{(n)}(t)$ hits zero with a direction to get negative. Now consider some time instance $t_i > t_1$. Then we will have $z_k^{(n)}(t) = -\min_{t_1\leq s\leq t}\left\{x_k^{(n)}(0) + A_k^{(n)}(s) - D_k^{(n)}(s)\right\}$. If $x_k^{(n)}(t_i) > 0$ then $x_k^{(n)}(0) - D_k^{(n)}(t_i) + A_k^{(n)}(t_i) > \min_{t_1\leq s\leq t_i}\left\{x_k^{(n)}(0) + A_k^{(n)}(s) - D_k^{(n)}(s)\right\}$, therefore $z_k^{(n)}(.)$ does not increase. On the other hand, if at t_i the scaled queue length process (with reflection term up to but not including t_i tends to become negative there is $x_k^{(n)}(0)-D_k^{(n)}(t_i)+A_k^{(n)}(t_i) < \min_{t_1\leq s<t_i}\left\{x_k^{(n)}(0) + A_k^{(n)}(s) - D_k^{(n)}(s)\right\}$, therefore $z_k^{(n)}(.)$ increases to $-(x_k^{(n)}(0) - D_k^{(n)}(t_i) + A_k^{(n)}(t_i))$ (it is an increase because in this case the expression inside the minimization is negative). For the cases where $x_k^{(n)}(t)$ hits the barrier at zero but immediately after it turns positive, the reflection does not change. We have thus shown that the reflection term (3.36) keeps indeed the queue lengths nonnegative and can increase only when the queue length hits zero.

Following the discussion in subsections 3.8.1 and 3.8.2, as all terms in the above expression converge weakly as $n \to \infty$, $z_k^{(n)}(t) \xrightarrow{w} z_k(t)$ accordingly, and actually becomes the amount of data that could have been transmitted in the asymptotic system if the queue was not empty at time t.

3.8.4 Further Analysis of the Equation

So far we have shown that under heavy traffic conditions a properly scaled version of the queue lengths converges weakly to

$$\mathbf{x}(t) = \mathbf{x}(0) - \int_0^t \mathbf{f}(\mathbf{u}(s))ds + \mathbf{\Sigma}_a \mathbf{w}_a(t) + \mathbf{\Sigma}_d \mathbf{w}_d(t) + \mathbf{z}(t).$$

By assumption, the arrival processes are independent of the channel processes so the corresponding Wiener processes are independent. Therefore, we have $\mathbf{\Sigma}_a \mathbf{w}_a(t) + \mathbf{\Sigma}_d \mathbf{w}_d(t) = \mathbf{\Sigma}\mathbf{w}(t)$, where $\mathbf{w}(t)$ is a standard K-dimensional Wiener process and $\mathbf{\Sigma}$ satisfying (3.9).

Chapter 4

Traffic Aware User Selection in MISO-OFDMA Downlink Systems with ZF precoding and TDD training

4.1 Introduction

The use of multiple antennas [4] has emerged as one of the enabling technologies to increase the performance of wireless systems. The ability to serve multiple users in the same time-frequency block has made the use of multiple antennas at the BSs particularly attractive for multiuser downlink systems, and the benefits coming from this fact are well understood [5]. In detail, in a downlink system where the BS has N antennas, at most N users can be scheduled simultaneously. The decision to be taken every timeslot then is (i) which users should be scheduled and (ii) how the corresponding signals should be precoded.

From an information theoretic point of view, the capacity region of the MIMO Broadcast Channel (BC) is well characterized [36], assuming perfect channel state information at the transmitter. However, achieving this region requires Dirty Paper Coding, which is complex to implement, while the assumptions of perfect channel state information and use of Gaussian codebooks are strong in practical systems. Linear precoding schemes such as ZF , a scheme that cancels the interference among the scheduled users, are more desirable to use in practice. There are many works on the issue of imperfect CSI , see for example [37] and references therein, [38, 39, 40], however the focus is mainly on quantities like sum throughput and they do not take into account the traffic processes of the users.

Since in practice the users of a wireless system request actual data, it is of interest to study the impact of MIMO in the higher layers [41]. For the MIMO MAC , a precoding strategy that achieves the stability region is presented in [42], under the assumptions of perfect CSI and use of Gaussian codebooks. This policy is based on Lyapunov drift minimization given the queue lengths and channels every timeslot and makes use of superposition coding and successive decoding. This is hard to implement in practice. Regarding the BC (i.e.the downlink system), authors in [43] have proposed a technique based on ZF precoding, with a heuristic user scheduling scheme that selects users whose channel states are nearly orthogonal vectors and illustrate the stability region this policy achieves via simulations. Authors in [44] notice that the policy resulting from the minimization of the drift of a quadratic Lyapunov function is to solve a weighted sum rate maximization problem (with weights being the queue lengths) each timeslot and they propose an iterative water-filling algorithm for this purpose. In addition, authors in [45] propose to use the delays of the packets in the head of each queue along with the queue lengths as weights. All these works assume accurate CSI available at the transmitter. In the case of delayed channel state information and channels having a correlation in time, authors in [46] compare the stability and delay performance of opportunistic beamforming and space time coding while in [47] they propose a user scheduling and precoding algorithm. In addition, in [48], the authors study the impact of channel state quantization in the stability of a system using ZF precoding under a centralized scheme where the transmitter selects the users to be scheduled based only on the queue lengths. However, the fact that radio resources i.e. time and/or spectrum are needed for the BS to acquire channel state information is not accounted for in these works.

In this chapter we consider a MISO downlink system where the BS acquires CSI from the users in TDD mode, in order to exploit the channel reciprocity. There are two ways for this: (i) users estimate their channel and then feeds back the CSI in a Time-Division Multiple Access (TDMA) manner and (ii) users send (pre-assigned) training sequences in the uplink so that the BS can estimate the channels (uplink and downlink channels are the same due to reciprocity). The latter scheme is implemented using orthogonal sequences among the users, so that the BS can decode every transmission without errors. Orthogonal sequences are produced e.g. by Walsh-Hadamard on pseudonoise sequences, and their length should be proportional to the number of users that simultaneously train in the uplink. Uplink training is considered the most promising for MIMO systems, since the length of the training sequences does not depend on the number of antennas at the BS . However, due to the orthogonality requirement, their length is proportional to the number of users that perform uplink training [1]. That means that in a system with many users, not all users should be selected to train at the same time, therefore the users that should train at every slot must also be selected. The TDD system model with uplink training has been also examined in [49], however they do not take into account that last observation, that is not all users should participate in the training at every slot. In this Chapter we focus on the tradeoff between having many users training (so having data transmitted to many users simultaneously) and having much time of the slot devoted to data transmission (which means having few users train). In order to simplify things, we focus on ZF precoding used at the transmitter. This scheme is widely used in the literature because it is simple to implement while capturing the fundamental tradeoffs arising from using multiple antennas and performing well in some scenarios of interest (e.g. in systems with many users [97] and/or with BSs with large antenna arrays). In addition, we will assume that all users that perform uplink training in a slot get scheduled. This is an assumption used fairly often in the literature concerning MIMO broadcast channels; in this context, the BS should select the set of "active users" at each timeslot and then transmit to them.

One natural approach would be to let the BS alone decide which users to schedule in every slot. This is the approach used in [48] and in current standards (e.g. LTE [11]), where the BS explicitly requests some users for their CSI . In the setting where traffic/queueing processes are considered, user selection can be done based on the statistics of the channels of the users and the state of their queue lengths at each slot. Unfortunately, using such centralized schemes, some scheduled users may have poor current channel states and some users with good channels may not be scheduled (i.e. may not feed back), which reduces the system performance. On the other hand, each user knows its own current channel state, and therefore decentralized feedback policies where the users decide based on their current channel states may improve the system performance.

[1] In the case where CSI acquisition is done by feedback in TDMA manner, then the time needed for CSI acquisition is also proportional to the number users that feed back.

This must be done properly as the decentralized policies require additional signalling information that may decrease drastically the improvement.

It is worth noting that recent works [50], [51] have shown that, in a network with simple physical layer (e.g. on-off channel, finite discrete channel states,...), decentralized algorithms like the recently proposed Fast CSMA [52] can achieve good performance. In addition, it has been shown in earlier works [53, 54] that up-to-date channel state information, which is known at the receivers, is more crucial than accurate queue length information, at least as far as stability is concerned. The scenario considered in this Chapter is more complicated as compared to the recent work on decentralized scheduling. In fact, in scheduling problems (e.g. OFDMA or TDMA), a user can directly estimate its bit rate using the current channel state. In multi-user MIMO systems, the bit rate of each user depends on the channel states of all users and the user cannot simply estimate its bit rate using its current channel state, which highly complicates the analysis.

In this Chapter, we examine three approaches to the user selection problem. The first one is centralized, in the sense that the transmitter decides which user will be scheduled (i.e. will train) at every slot. The second approach, which we term as "decentralized", is to let the users decide which of them should actually feed back via some contention/coordination scheme. The main idea behind this approach is that every user can know its channel state, therefore a user with a very bad channel state will choose not to feed back (contrary to what can happen in the centralized approach). More specifically, in this case, the transmitter specifies the number of users to be scheduled and lets the users decide in a decentralized manner who will be the ones that will actually get scheduled in the slot. Combined with some (infrequent) signalling regarding the users queue lengths from the BS , we prove that properly combining the decentralized and centralized approaches leads to a bigger achievable stability region than using the centralized approach alone. Detailed description of these policies is given in Section 4.3, after a presentation of the system model in Section 4.2. Section 4.4 presents some calculations regarding the rate distributions and some general intermediate results regarding stability, that will be used for the proofs in subsequent Sections. In Section 4.5 we examine in detail a special case, namely the $2-$user system with i.i.d. channels and single rate level. This is a case where the stability regions can be expressed in close form and plotted, and helps illustrate why combining a decentralized and a centralized approach helps in enlarging the stability region of the system. After that, Section 4.6 contains stability analysis in the general case of K users, while extensions to the case where multiple channels are used (e.g. via OFDMA modulation) is covered in Section 4.7. In the latter Section, we also prove that in the case of a transmitter with a single antenna, the decentralized policy can achieve a very large fraction of the stability region achieved in the ideal case with full channel state information at no cost at the transmitter, with no need of the channel statistics. Finally, in Section 4.8 we discuss an alternative implementation where extra signalling bandwidth instead of time is used for the control signals required to be broadcasted for the decentralized/mixed policies and Section 4.9 presents a threshold-based policy in the cases where continuous time for contention is not possible. The proofs for the derivations of the stability regions are done based on the method of first proving that the stated region is achievable by a rule that does not take into account the queue lengths, prove, using the Foster-Lyapunov criterion, that the proposed policy achieves at least as big region as the first rule and then prove that there is no policy achievng more than the stated region. This method was first used in [13] and then in many other works dealing with scheduling and stability. See, for example, [15] (where the rule to prove the converse is termed $\omega-$ only policy) and [55], [56] (where the scheduling rule to prove the converse is referred to as Static Service Split rule), where the fluid asymtotics of the system are examined. Finally, Section 4.10 concludes the Chapter.

4.2 System Model

4.2.1 Physical Layer Model

We consider a single cell wireless system serving K users with N antennas at the BS . The users are equipped with a single antenna each. Time is slotted. Each channel is i.i.d. Rayleigh block fading, i.e. the channels stay constant in a slot of T_s channel uses and change independently in the next slot. The channel of user k can be written as an N-dimensional complex vector $\mathbf{h}_k(t) = \sqrt{\bar{g}_k}\hat{\mathbf{h}}_k(t)$ where $\hat{\mathbf{h}}_k(t) \sim \mathcal{CN}(0, \mathbf{I}_N)$ and $\bar{g}_k$ represents the channel gain due to large scale fading and is assumed to be constant (e.g. following a path loss model depending on the distance from the BS). In this case, the channel vector can be also written as $\mathbf{h}_k(t) = \sqrt{g_k(t)}\mathbf{u}_k(t)$, where $\mathbf{u}_k(t)$ is an isotropically distributed unitary vector and $g_k(t) = ||\mathbf{h}_k(t)||^2$ is the channel magnitude. We have the following:

Lemma 4.2.1. *The channel magnitude of user k is distributed with CDF :*

$$\mathbb{P}\left\{g_k(t) < x\right\} = \frac{\gamma\left(N, \frac{x}{2\bar{g}_k}\right)}{\Gamma\left(N\right)}. \tag{4.1}$$

Proof. We can write $g_k(t) = \bar{g}_k \left|\left|\hat{\mathbf{h}}_k(t)\right|\right|^2$, therefore the second term has the chi-squared distribution with $2N$ degrees of freedom. We thus have

$$\mathbb{P}\left\{g_k(t) < x\right\} = \mathbb{P}\left\{\left|\left|\hat{\mathbf{h}}_k(t)\right|\right|^2 < \frac{x}{\bar{g}_k}\right\} = \frac{\gamma\left(N, \frac{x}{2\bar{g}_k}\right)}{\Gamma\left(N\right)}, \tag{4.2}$$

which is the stated result. □

Since the BS is equipped with multiple antennas, multiple users can be served simultaneously by precoding the corresponding transmit signals. In this Chapter, we consider ZF precoding, i.e. a linear precoder such that the intracell interference caused to any user that is served is zero. The choice is motivated by the relatively low complexity of linear precoders (and ZF in particular) with respect to the non-linear ones and by the fact that Zero Forcing is a simple scheme to analyze, that however captures the fundamental tradeoffs in multiple antenna transmission and has very good performance in many scenarios of interest (as, for example, in the case where large antenna arrays are used).

As for any linear precoder $\mathbf{W}(t) = [\mathbf{w}_1(t), ..., \mathbf{w}_K(t)]$, the signal received by user k at slot t is

$$y_k(t) = \mathbf{h}_k^H(t)\mathbf{w}_k(t)s_k(t) + \sum_{j\neq k} \mathbf{h}_k^H(t)\mathbf{w}_j(t)s_j(t) + n_k(t) \tag{4.3}$$

where $s_j(t)$ is the data symbol intended to user j, assumed complex Gaussian with zero mean and unit power, $n_k(t) \sim \mathcal{CN}\left(0, \sigma^2\right)$ is the white noise at the receiver or user k. The BS has available transmission power P, that is $\mathrm{tr}(\mathbf{W}\mathbf{W}^H) \leq P$. The achievable rate of user k at slot t is $r_k(t)$ (bits/channel use), which depends on the corresponding Signal-to-Noise Ratio (SNR) at time t. We will assume that the achievable rates can take values from the set $\mathcal{R} = \{R_1, ..., R_l, .., R_L\}$, with $R_1 = 0$, $R_{l-1} < R_l$ and $R_L < \infty$; this is the case in practice as a finite number of modulation and coding schemes are used for transmission. In addition, we assume that the possible rate levels are known to the BS and mobiles, which is rather reasonable since they are specified by the communications protocol used. Also we assume that rate R_l can be supported if the SINR at the receiver is above some appropriately defined threshold S_l.

Under this model, let $\mathcal{F}(t)$ be the set of users that are scheduled at slot t, $F(t) = |\mathcal{F}(t)|$ and $k(1), .., k(i), ..., k(F(t))$ the corresponding permutation of user indices. Also, define

$$\mathbf{H}_{k(i)}(t, \mathcal{F}) = [\mathbf{h}_{k(1)}(t), ..., \mathbf{h}_{k(i-1)}(t), \mathbf{h}_{k(i+1)}(t), ..., \mathbf{h}_{i(F(t))}(t)].$$

To further reduce the complexity of the transmission scheme, the total transmit power is split equally to each of the users scheduled. Then, the precoding vector for user $k(i), \forall i \in \{1, .., F(t)\}$ is given as the projection of the channel of this user on the nullspace generated by the channels of the other users:

$$\mathbf{w}_{k(i)}(t) =$$
$$\sqrt{\frac{P}{F}} \frac{\left(\mathbf{I}_N - \mathbf{H}_{k(i)}(t, \mathcal{F})(\mathbf{H}^H_{k(i)}(t, \mathcal{F})\mathbf{H}_{k(i)}(t, \mathcal{F}))^{-1}\mathbf{H}^H_{k(i)}(t, \mathcal{F})\right)}{\left|\left|\left(\mathbf{I}_N - \mathbf{H}_{k(i)}(t, \mathcal{F})(\mathbf{H}^H_{k(i)}(t, \mathcal{F})\mathbf{H}_{k(i)}(t, \mathcal{F}))^{-1}\mathbf{H}^H_{k(i)}(t, \mathcal{F})\right)\mathbf{h}_{k(i)}(t)\right|\right|} \mathbf{h}_{k(i)}(t). \tag{4.4}$$

The corresponding SNR (since interference is suppressed) for this user will then be

$$\begin{aligned} \mathrm{SNR}_{k(i)}(t) = \frac{P \left|\left|\mathbf{h}_{k(i)}(t)\right|\right|^2}{\sigma^2 F(t)} \mathbf{u}^H_{k(i)}(t) \Big(\mathbf{I}_N \\ - \mathbf{H}_{k(i)}(t, \mathcal{F}(t))(\mathbf{H}^H_{k(i)}(t, \mathcal{F}(t))\mathbf{H}_{k(i)}(t, \mathcal{F}(t)))^{-1}\mathbf{H}^H_{k(i)}(t, \mathcal{F}(t)) \Big) \mathbf{u}_{k(i)}(t). \end{aligned} \tag{4.5}$$

From the above, it can be seen that in order to transmit using Zero Forcing, accurate channel state information of the channels of the users that are scheduled is needed. This information is not available to the BS and must be acquired by using feedback or training from the receivers. For consistency, we will consider the case where CSI is acquired by uplink training from the users. This means that channel estimation is done in TDD mode, exploiting reciprocity; this is a promising approach, especially for large antenna arrays at the BS , since the feedback overhead does not scale with the number of antennas. It does scale with the number of users that train however, meaning that when CSI is acquired by too many users there will be little time left to transmit in the timeslot before the channels change again: This problem is exactly the focus of the Chapter. On the other hand, even if CSI is acquired by feedback in FDD mode, the BS must wait for the feedback from the users to be received before precoding [38]. Also under our model of i.i.d. block fading channels, outdated feedback is not useful. The above imply that the main ideas and results of the analysis presented can be useful even in systems with feedback in FDD mode, assuming accurate channel estimation from the users' side, enough bit rate in the reverse link for perfect CSI in the BS after the feedback procedure and that the bandwidth in the uplink is not enough for all users to feed back simultaneously in parallel channels. In addition, we will assume throughout this Chapter that there are no errors in the channel estimation, in order to focus on the impact of time needed for training in our system. In practice this assumption can hold if pilot sequences of very high power are used.

4.2.2 Queuing model and impact of training

Each of the K users in the cell has an incoming traffic process $a_k(t)$, which is an integer-valued process, measured in bits, i.i.d. in time and independent across users with $a_k(t) < A_{max}$ almost surely for some finite constant A_{max}. This quantity is assumed to be known to the scheduler and users. The mean rate of this process is $\mathbb{E}\{a_k(t)\} = \lambda_k$. Data for user k is stored in a respective buffer until transmission and let $q_k(t)$ denote its size in bits at the beginning of slot t.

Denote now $z_k(t)$ as the schedule in timeslot t, that is $z_k(t) = 1$ if user k is scheduled for this timeslot (i.e. if user k has actually reported its channel to the BS). We are under the constraints that (i) $F(t)$ users are scheduled at each timeslot, with $F(t) \leq N$ (ii) for every channel the BS schedules a user whose channel state is known at the maximum possible rate it can support (that is ensuring transmission without errors). In addition, we will denote here $\tau(t)$ the number of channel uses used for training and signalling in the slot t. This means that data is transmitted for a scheduled user for $(T_s - \tau(t))$ channel uses, therefore, if the rate supported to user k at timeslot t is $r_k(\mathbf{W}(t), \mathbf{H}(t)) \in \mathcal{R}$ bits per channel use, the corresponding service process will be $(T_s - \tau(t)) r_k(\mathbf{W}(t), \mathbf{H}(t))$ bits [2]. The queues then evolve as follows, $\forall k \in \{1, .., K\}$:

$$\begin{aligned} &q_k(t+1) = \left[q_k(t) - \lfloor (T_s - \tau(t)) r_k(\mathbf{W}(t), \mathbf{H}(t)) \rfloor z_k(t)\right]^+ + a_k(t), t \geq 0 \\ &q_k(0) = a_k(-1). \end{aligned} \tag{4.6}$$

In the above, $a_k(-1)$ is defined as a random variable drawn from the distribution of $a_k(t), t \geq 0$. This constraint actually means that we start measuring time after the first arrivals in the queues so that the queues do not start empty (and the broadcast of the queue lengths at time $t = 0$ not to be the zero queue). This is done for more convenience in analysing the proposed algorithm, however, since we are interested in the case when the system is left running for too long (i.e. $t \to \infty$) and the arrival processes are bounded, the choice of the initial condition does not really affect our results.

We are interested in the stability of the system, in the sense discussed in Section 2.2.2 (i.e. strong stability). Equation (4.6), thus, implies that training affects essentially the service rate, and thus the stability region, in two ways: First, more time devoted to training leads to lower service rate for the users actually scheduled in the timeslot. On the other hand, if more users participate in the training, more users can get scheduled in a timeslot, thus overall a user can get higher mean service rate. The focus of this Chapter is, then, this tradeoff

[2]the arguments in the rate stress that it depends on the channel state and precoder at timeslot t

and how to efficiently design user selection strategies to achieve large stability regions.

For simplicity, we only consider schemes where all users whose CSI is acquired get scheduled. Also, as mentioned before, transmit power is allocated equally to scheduled users. The first assumption is common in the literature concerning MIMO broadcast systems, where all "active users" are scheduled, see e.g. [38]. In addition, even in the case of full CSI without any cost, the highest stability region would be given from solving a weighted sum maximization problem in each timeslot (in accordance to the MaxWeight rule [13, 27]). Joint scheduling and power control even in this ideal case is a hard problem, especially in our model with finite rate set (some algorithms like iterative waterfilling [44] have been proposed but using the information theoretic capacity for the service rates). Adding the cost of feedback on top of it would make the problem more complicated and result in a solution of high computational complexity (see e.g. [66] for the problem in the single antenna case, where only approximations of the optimal solution are implementable in practice). The problem then reduces to finding strategies to choose the "active" users at each time slot.

4.3 Proposed Policies for User Scheduling

In this section we present in detail the scheduling and training policies to be analyzed in the present work. Before proceeding in the descriptions, we define R_0 (bits per channel use) the rate at which the control information from the BS to the users can be broadcasted. Further, we assume that when F users perform the uplink training, they use orthogonal pilot of length βF channel uses each. β is an integer system parameter, not smaller than 1 (for the pilots to be indeed orthogonal). Greater length of training sequences implies that the training symbols should have less power (for the same quality of estimation) so this parameter can be tuned according to the power capabilities of the user terminals. Finally, downlink pilots are assumed of a length of β_p channel uses. Since at least a downlink pilot and the uplink pilots must be used, the maximum number of users to be scheduled is the maximum number of users that can participate in training within the duration of the timeslot, that is

$$F_{max} = \min\left\{N, \left\lfloor \frac{T_s - \beta_p}{\beta} \right\rfloor\right\}. \tag{4.7}$$

In practice this number may be actually lower, for example it is usually desirable that at least half of the timeslot is used for data transmission [6].

4.3.1 Centralized policy

Since the channel statistics are known, one approach is to let the BS decide which users to acquire CSI from and schedule based on the expectation of the rate they receive. Each expectation is found over the joint probability distribution of the channel realizations of *all* users and its expression is given in Section 4.4.1. For this scheme, the BS sends a downlink pilot to allow the users to estimate their channel and decode the control messages. After the pilot, there is a control phase, where the BS broadcasts the Identification Numbers (IDs) of the users that will get scheduled at this timeslot. For each user selected

$$\beta_c = \frac{\log_2 K}{R_0}$$

channel uses are needed. In addition, there must be a way for the users to know that the control part is over and that those among them that are scheduled should start training. Here we propose that the signal for that is theBS staying silent for one channel use. An alternative, and perhaps more robust, way would be to assign a corresponding sequence. However we chose the one channel use of staying silent scheme for simplicity and because it will give an upper bound on the performance of the centralized scheme [3], thus the worst case for the improvement achieved with the decentralized and mixed schemes that follow. Notice that it is a function of the total number of users admitted in the cell and the rate for the control signalling. It can thus pose some limitation in the number of users admitted in the cell (the time used for signalling in each slot should not be too big). The users that are selected then perform uplink training and then theBS serves then using ZF precoding with equal power among users, as explained in the previous Section. This procedure is illustrated in Fig. 4.1.

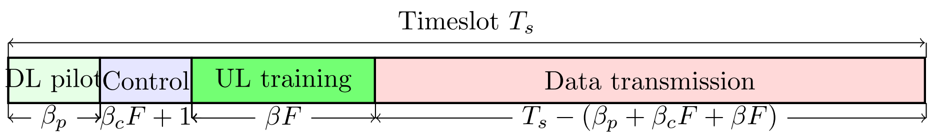

Figure 4.1: Operation of the centralized scheme in a timeslot where F users have been scheduled

Alternatively, if the control phase is to remain constant irrespective of the number of scheduled users, the control phase will last for $\frac{K}{R_0}$ channel uses instead of $\beta_c F$, because a codeword of K bits, one for every user indicating if he is

[3]Since the control channel must be decoded successfully at all times, a lower rate of one bit per channel use may be needed.

scheduled or not, should be used. This poses more severe restrictions to how many users the cell can support but having a control region of fixed duration may be desirable in practice e.g. for synchronization purposes.

The BS selects the set of users to be scheduled at every slot as the solution to the following problem:

$$\mathcal{F}(t) = \arg\max_{\mathcal{F}\in 2^{\mathcal{K}}} \left\{ (T_s - (1 + \beta_p + \beta_c F + \beta F)) \sum_{k\in\mathcal{F}} q_k(t)\mathbb{E}\left\{r_k(t)|F\right\} \right\}, \quad (4.8)$$

where the expectation is taken with respect to the joint probability distribution of the channels, as presented in detail in Section 4.4.

The advantage of this scheme is its (relative) simplicity. Indeed, the expectations of the rate for every user k given that $F-1$ other users are scheduled can be computed in advance and used at every slot. Furthermore, if the channels are i.i.d. among the users, it can be implemented by having F run from 1 to F_{max}, sort the users according to the values of $q_k(t)\mathbb{E}\left\{r_k(t)|F\right\}$ and select the F biggest every time. In the end select the configuration that gave the biggest expected weight. The overall computational complexity here is $O(F_{max}K\log_2(K))$.

The downside is that the actual realizations of the channels are ignored; for instance, a user chosen to be scheduled may actually have a very bad channel (i.e. channel with such a bad magnitude that cannot even support the smallest rate). This is a bigger problem when OFDMA is employed (as is actually done in modern systems e.g. LTE and Worldwide Interoperability for Microwave Access (WiMax)) because according to this scheme the same users will be scheduled for every carrier, so the frequency diversity in the fading is not exploited.

4.3.2 Decentralized policy with periodic signalling

To overcome the shortcomings of the centralized scheme, we first note that each user *can* know its actual channel realization, namely via downlink training. In this case, if each user knew its queue length (or the ranking of users based on the queue length) as well, we could exploit this knowledge and use, for example, techniques inspired by queue-based CSMA [98] or Fast CSMA [52] to find a schedule. Indeed, it has been recently shown that performing a CSMA with the backoff timer being a function of the product of the queue length times the actual rate supported by the channel realization can achieve throughput optimality in uplink systems with single carrier and single antenna fading channels [51] (under the assumption of continuous time for backoff). However, in our case the system model is more complicated because the users do not know their queue lengths and because of multiple antennas in the BS , more than one user can be scheduled simultaneously. Our proposed schemes, detailed in the next paragraph, are based on two ideas: (i) the BS periodically broadcasts the (suitably quantized) values of the queue lengths and (ii) the BS decides on the *number* of users to be scheduled and based on that lets the users contend using the queue length information they have and an estimate of their achievable rate based on channel state realization.

4.3.2.1 Algorithm description

To begin with, every T timeslots, that is at time $0, T, 2T, ..., mT, ...$ the BS broadcasts quantized versions of the queue lengths of the users at the beginning of this slot, i.e. broadcasts a quantization of the vector $\mathbf{q}(mT), m = 0, 1, ...,$ the restriction being that T is a finite number. The quantization of the queue lengths is discussed in detail in Section 4.3.2.2. In addition, the BS broadcasts the number $F(mT)$ of users to report the channel each timeslot for the next $T-1$ consecutive timeslots. No data transmission at this timeslot takes place in order to make broadcasting this information possible (with the BS adopting e.g. uniform precoding for transmission).

Denote now by $\hat{\mathbf{q}}(t) := \tilde{\mathbf{q}}(T\lfloor \frac{t}{T} \rfloor)$ to be the most recent information about their queue state that the users have. At each timeslot the BS sends a downlink pilot with duration β_p channel uses so that the users can estimate their channels and lets a period of τ channel uses for the users to contend for channel access. Assuming that contention can be done in continuous time and with signals of negligible duration (this assumption has been implicitly used in recent works dealing with Fast CSMA over fading channels [52], [50]), user k waits until time

$$\tau_k' = \frac{\tau_c'}{\hat{q}_k(t)\mathbb{E}\{r_k(t)|g_k(t), F(t)\}}. \tag{4.9}$$

In the above equations, the times are expressed in time units (i.e. ms or μs). The denominator is the latest broadcasted value of the queue length of this user times the expectation of the rate the user will get if it is scheduled, given its own channel realization (we have defined here $g_k(t) = ||\mathbf{h}_k(t)||^2$). This computation is detailed in the Section 4.4.1, and for an environment with Rayleigh fading (the case we examine here) can be done in a totally decentralized manner. Note that under this scheme, the F users with the biggest values of $\hat{q}_k(t)\mathbb{E}\{r_k(t)|g_k(t), F(t)\}$ are the ones that get actually scheduled.

Once the contention period is over, the F first users to have a signal broadcasted feed back their IDs in a TDMA manner, e.g. in the sequence in which they sent the contention signals (under the assumptions on continuous time contention and very short signals the users can know their place in the sequence).

These users then perform uplink training and the BS transmits to them using Zero Forcing. The total time for transmission is then $(T_s - \beta_p - \tau - (\beta + \beta_c)F(m))$ channel uses. Illustration of a timeslot under this policy is given in Fig. 4.2.

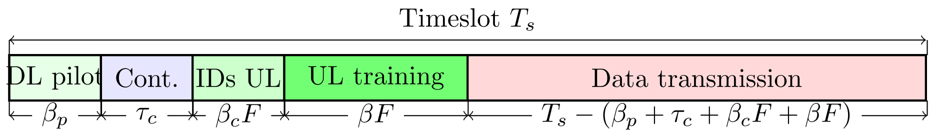

Figure 4.2: Operation of the decentralized scheme in a data timeslot where the Base Station has signalled that F users are to be scheduled

What remains to be specified is the number of users to feed back in every period $\{mT + 1, ..., (m-1)T\}$. As stated before, this decision is taken in the beginning of timeslot mT, based on the corresponding queue length information and channel statistics. Here we consider that the number of users to get scheduled is given as the solution of the following problem.

$$\begin{aligned} F^*(m) =& \arg \max_{F=1,..,F_{\max}} \Bigg\{ (T_s - \beta_p - \tau_c - (\beta + \beta_c)F) \\ & \mathbb{E}_{\mathbf{g}(t)} \Bigg\{ \max_{\mathcal{F}:|\mathcal{F}|=F} \sum_{k \in \mathcal{F}} \hat{q}_k(mT) \mathbb{E}\left\{ r_k(mT) | g_k(t), F \right\} \Bigg\} \Bigg\}. \end{aligned} \tag{4.10}$$

In the above, the outer expectation is with respect to the joint distribution of the channel magnitudes while the inner expectation is with respect to the joint distributions of the channel directions (for the channels of all users in the system). A way to do these calculations is to use the recently proposed framework in [99], [100] for partial sums of order statistics of non-identically distributed ransom variables.

4.3.2.2 Queue length quantization scheme

As noted in the description of the algorithm, the BS broadcasts quantized versions of the queue lengths of the users. This quantization is essential because, as noted in the beginning of the Section the rate at which the BS can broadcast signalling information is R_0 bits per channel use; this means that if a slot is used for signalling, at most $T_s R_0$ bits can be sent to the users. Given that the BS should broadcast K queue lengths and how many users are to feed back, the number of bits b_q used for quantization of each queue should satisfy the following:

$$K b_q + \log_2 F_{max} \leq T_s R_0. \tag{4.11}$$

The above inequality poses one limitations as to how many users can be supported by the system if it operates under this scheduling policy and a limitation to the accuracy of the queue length feedback for a given number of users in the cell. However, if multiple antennas and carriers are used, this limitation is not very severe.

We now detail the way the queue lengths are actually quantized for a given number of bits per user, b_q. To this end, we define

$$Q = \max\{TR_L, TA_{max}\} \tag{4.12}$$

and the intervals $[0, Q], [Q, 2Q], ..., [(p-1)Q, pQ], ...$ Note that Q is the biggest change that can possibly happen to a queue length after T slots (the queue will decrease by at most TR_L-if at every slot is served at the maximum rate with no further arrivals- and increase by at most by TA_{max}-if it s not served at all and has the maximum possible arrivals at each slot). Therefore, every T slots each queue length will be in one of the aforementioned intervals, and furthermore, given that at mT a queue length was at the p-th interval, at $(m+1)T$ it will be at intervals $p-1$, p or $p+1$. The idea is then to set the quantization interval to $[0, Q]$ at the beginning and inform each user every T slots if it stays in the same interval of one of the neighbouring intervals (this can be done at a signalling as low as 2 bits per user or even $1.5K$ bits in total). Then, the quantized queue length is sent, assuming uniform quantization within each interval using the rest of the signalling bits available. More concretely, if b_q bits per user are used for quantization 2 bits are used to denote the quantization interval and the rest are used to point to one of the 2^{b_q-2} levels within the interval. If $b_q = 2$, then the middle value of the interval is used as an estimation of the queue length. Note that this way, the difference between each of the broadcasted queue lengths, denoted by $\tilde{\mathbf{q}}(mT)$ and the corresponding real queue length from $\mathbf{q}(mT)$ is at most Q .

Finally, some remarks are in order. First, we have assumed that the control information broadcasted by the BS are always decodable at the user terminals. This is a rather frequent and reasonable assumption in the literature (i.e. that signalling and control data are transmitted without errors). In practice, control data are transmitted using low rate modulations and strong coding schemes. We can also always assume that we do downlink power control for the signalling information. In addition, if the number of carriers and antennas are high, the diversity of the system is so big that control data can always be received successfully even if some channels of the users are in deep fade (also, users with very bad average channel conditions should not be admitted into the cell). Accurate estimation of the users channel can be similarly argued by using high power pilots.

4.3.3 Mixed Policy: Combining Centralized and Decentralized Schemes

The decentralized approach to the user selection problem should lead to users with better channel conditions being selected in general, however it requires some extra time overhead for the contention period. In some cases, some queues may be empty or a few queues may be much bigger than others. If this happens it may be better for the users with the much bigger queues to be scheduled for the $T-1$ timeslots without any additional signalling overhead. Note that since the same users get scheduled every $T-1$ consecutive slots, no overhead for their IDs must be used either. The operation of this scheme in a data timeslot is illustrated in Fig. 4.3 [4]. We will refer to this scheme as "periodic centralized policy" for the rest of the Chapter.

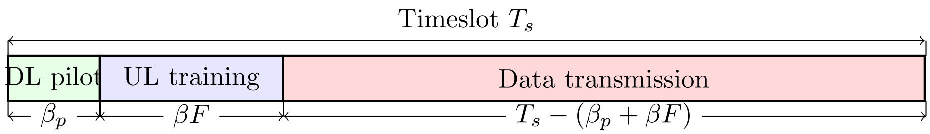

Figure 4.3: Operation of centralized scheme with periodic user selection in a data timeslot where the Base Station has signalled a set of F users to be scheduled

The set of users to schedule according to this periodic centralized policy is set as

$$\mathcal{F}^*(m) = \arg\max_{\mathcal{F}\in 2^{\mathcal{K}}} \left\{ (T_s - (\beta_p + \beta F)) \sum_{k\in\mathcal{F}} \hat{q}_k(mT)\mathbb{E}\left\{r_k(t)|F\right\} \right\}. \tag{4.13}$$

The mixed policy here is, therefore, that the BS at every slot $t = mT, m = 0, 1, ...$ decides that, for the next $T-1$ slots, either the decentralized policy will be used, with the optimal number of users to get scheduled as given by eq. (4.10) or select a set $\mathcal{F}(m)$ of users to schedule according to the maximization in eq. (4.13). This decision is made based on which of the two policies will maximize the quantity $\mathbb{E}\left\{\sum_{k=1}^{K} \hat{q}_k(mT)(T_s - \tau(t))r_k(t) \middle| \mathbf{q}(mT)\right\}$, with the expectation taken over the channel distributions (which affect the outcomes of the policies). More concretely, let $F^*(m)$ be given by (4.10) and $\mathcal{F}^*(m)$ be given by (4.13).

[4] The DL pilot is still needed for the users to set the power of their training sequences such that the SNR in the reverse link suffices for perfect estimation

Then, the BS selects the decentralized scheme with $F^*(m)$ users to get scheduled if

$$(T_s - \beta_p - \tau_c - (\beta + \beta_c))\mathbb{E}_{\mathbf{g}(t)}\left\{\max_{\mathcal{F}:|\mathcal{F}|=F^*(m)} \sum_{k\in\mathcal{F}} \hat{q}_k(mT)\mathbb{E}\left\{r_k(mT)|g_k(t), F\right\}\right\}$$
$$\leq (T_s - (\beta_p + \beta F)) \sum_{k\in\mathcal{F}^*(m)} \hat{q}_k(mT)\mathbb{E}\left\{r_k(t)||\mathcal{F}^*(m)|\right\}$$

and the centralized scheme with the users in $F^*(m)$ otherwise.

This policy has a bigger stability region than either of the two policies mentioned before in this Section, since it essentially combines both. It needs 1 additional bit of signalling compared to the other policies, in order to inform the users if the centralized or decentralized scheme will be employed. In addition, the BS needs to broadcast the number F of users to be scheduled in the decentralized policy if used ($\log_2(F_{max})$ bits) or the users to get scheduled in case the centralized policy is used ($\min\{K, F_{max}\log_2 K\}$ bits). Since $F_{max} \leq K$, there must hold

$$Kb_q + \min\{K, F_{max}\log_2 K\} \leq T_s R_0, \tag{4.14}$$

which gives a bound on the number of users to be admitted to the cell in order to operate under the mixed policy.

4.4 Calculation of Parameters and Stability Results

In this Section we give the expressions for the SNR (and subsequently rate) distributions. Also, we give some useful lemmas about stability of systems where control decisions are done periodically and not in a slot-per-slot basis.

4.4.1 Calculation of average rates

The decentralized scheme requires that every user should calculate their average rate given their current channel state realization, as seen by eq. (4.9). Indeed, since the system operates under isotropic fading directions, we can calculate the probability distribution over the other users' channels and zero forcing precoding of a user's SNR given its channel. We have:

Proposition 4.4.1. *The probability that the received SNR at user k exceeds s given its channel strength and that this user and other $F-1$ users are scheduled* *is given by*

$$\begin{aligned}\mathbb{P}\left\{SNR_k(t) > s|g_k(t), F\right\} &= \mathbb{P}\left\{SNR_k(t) > s|\mathbf{h}_k(t), F\right\} \\ &= 1 - I_B\left(\frac{F\sigma^2}{Pg_k(t)}s; N-F+1, F-1\right).\end{aligned} \tag{4.15}$$

This distribution is with respect to the direction of the channel of user k and the channels of the other users that get scheduled.

Proof. Please refer to Section 4.11.1 in the Appendix for this Chapter for the proof. □

From the above result and the proof we can see that only the magnitude and not the direction of the channel realization comes into the equation. In addition, the long-term statistics of the other users do not play any part in the computation either. Intuitively these remarks are due to the isotropic direction of the channel vectors. Indeed, since we are considering ZF precoding, the loss of SNR comes due to the fact that the channels are not orthogonal, therefore the demand of causing zero interference can constrain a lot the precoder selection. Since the directions are isotropic, knowledge of one channel direction does not imply anything about how nullspace of the other users should behave. A consequence of these remarks is that a user can actually calculate this distribution (and hence the average rate he will get given its channel) with only the knowledge that the whole system operates under Rayleigh fading.

In the centralized scheme, where the BS does not have knowledge of the magnitude realization of the channels, the probability that the SNR of user k exceeds S when $F-1$ other users are scheduled is the following [48, 101][5]

Proposition 4.4.2. *Given a number of users to be scheduled, F, the probability that the SNR of user k exceeds S is given as*

$$\mathbb{P}\left\{SNR_k(t) > s|F\right\} = 1 - \frac{\gamma\left(\frac{F\sigma^2}{\bar{g}_k P}s; N-F+1\right)}{\Gamma(N-F+1)}.$$

From the results of Propositions 4.4.1 and 4.4.2 we can find the average rates conditioned on the channel realization of user k and the expected rate of this user without knowing the channel realization, respectively. More concretely, if we define

$$L_{0,k}(t, F) = \max\left\{l \in 1, ..., L : S_l \leq \frac{g_k(t)P}{F\sigma^2}\right\}, \tag{4.16}$$

[5]It can also be calculated by integrating (4.15) for $g_k(t)$ from zero to infinity. Although we could not obtain the exact form given in Proposition 4.4.2, numerical results indicate that the numerical values are the same with either method. We will use the latter expression since it is in a closed form

i.e. the index of the highest rate that could be supported by user k if he is scheduled and his channel is orthogonal to the channels of the other $F-1$ scheduled users, we have:

$$\mathbb{E}\left\{r_k(t)|F, g_k(t)\right\} = \sum_{l=1}^{L_{0,k}(t,F)} R_l \mathbb{P}\left\{S_l \leq SNR_k(t) < S_{l+1}|F, g_k(t)\right\}$$
$$= \sum_{l=1}^{L_{0,k}(t,F)} R_l \left(\mathbb{P}\left\{SNR_k(t) \geq S_l|F, g_k(t)\right\} - \mathbb{P}\left\{SNR_k(t) \geq S_{l+1}|F, g_k(t)\right\}\right). \tag{4.17}$$

and

$$\mathbb{E}\left\{r_k(t)|F\right\} = \sum_{l=1}^{L} R_l \left(\mathbb{P}\left\{SNR_k(t) \geq S_l|F\right\} - \mathbb{P}\left\{SNR_k(t) \geq S_{l+1}|F\right\}\right). \tag{4.18}$$

Notice that, since the statistics are assumed known, the rates in (4.18) can be calculated only once and used by the BS for the centralized scheme.

4.4.2 Stability results

In this subsection we are interested in deriving some stability results for the system under the policies where slots $\{0, T, T+1, .., mT, ..\}$ are used for signalling and/or broacasting of the queue lengths. First, we define the queueing system that results when examining the original system at time instances $0, T, ...$, i.e. at the beginning of the slot in which the broadcasting takes place. Formally:

$$\tilde{\mathbf{q}}(m) := \mathbf{q}(mT), m = 0, 1, 2, ... \tag{4.19}$$

The equations regarding the evolution of this system are, thus $\forall k \in \{1, ..., K\}$:

$$\tilde{q}_k(m+1) = \tilde{q}_k(m) + \sum_{t=0}^{T-1} a_k(mT+t) - \sum_{t=1}^{T-1} z_k(mT+t)\mu_k(mT+t) + \sum_{t=1}^{T-1} y_k(mT+t) \tag{4.20}$$

where $z_k(t)$ is the indicator function, set to 1 if user k is scheduled in timeslot t and zero otherwise, $\mu_k(t)$ is the total number of bits assigned for transmission to user k at timeslot t (that is $\mu_k(t) = r_k(t)(T_s - \tau(t))$ (recall that $\tau(t)$ is the total time of the slot used for pilot transmission, coordination and training), and $y_k(mT+t) = [z_k(mT+t)\mu_k(mT+t) - q_k(mT+t)]^+$ the number of "wasted" bits if the offered rate at one timeslot is bigger than the available bits in the buffer. Note that the process $\tilde{\mathbf{q}}(m)$ is a discrete time Markov chain evolving on a countable state space. The following result holds:

Lemma 4.4.1. *The system $\mathbf{q}(t)$ is strongly stable if and only if the system $\tilde{\mathbf{q}}(m)$ is strongly stable.*

Proof. Assume first that $\mathbf{q}(t)$ is strongly stable. We have $\frac{1}{M}\sum_{m=0}^{M-1}\mathbb{E}\{\tilde{q}_k(m)\} \leq T\frac{1}{MT}\sum_{\tau=0}^{T(M-1)}\mathbb{E}\{q_k(\tau)\}$ therefore

$$\lim_{M\to\infty}\sup\frac{1}{M}\sum_{m=0}^{M-1}\mathbb{E}\{\tilde{q}_k(m)\} \leq T\lim_{M\to\infty}\sup\frac{1}{MT}\sum_{\tau=0}^{T(M-1)}\mathbb{E}\{q_k(\tau)\} < +\infty,$$

where the second inequality follows from the assumption. Therefore $\tilde{\mathbf{q}}(m)$ is indeed strongly stable. Assume now that $\tilde{\mathbf{q}}(m)$ is strongly stable. We can write

$$\begin{aligned}\lim_{t\to+\infty}\sup\frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\{q_k(\tau)\} &= \lim_{M\to+\infty}\sup\frac{1}{MT}\sum_{\tau=0}^{MT-1}\mathbb{E}\{q_k(\tau)\}\\ = \lim_{M\to\infty}\sup&\left(\frac{1}{MT}\sum_{m=0}^{M-1}\mathbb{E}\{q_k(mT)\} + \frac{1}{MT}\sum_{m=0}^{M-1}\sum_{\tau'=1}^{T-1}\mathbb{E}\{q_k(mT+\tau')\}\right).\end{aligned} \tag{4.21}$$

Since $\tilde{\mathbf{q}}(m)$ is strongly stable and $\tilde{\mathbf{q}}(m) = \mathbf{q}(mT)$, there exists some $0 < C_0 < \infty$ such that $\forall k \in \{1, .., K\}$:

$$\lim_{M\to\infty}\sup\frac{1}{MT}\sum_{m=0}^{M-1}\mathbb{E}\{q_k(mT)\} \leq \frac{C_0}{T}. \tag{4.22}$$

Also, note that $\forall\tau' \in \{1, .., T-1\}$, $\forall m = 0, 1, ...$ it holds $q_k(mT+\tau') \leq q_k(mT)+\tau' A_{max}$. This implies that, $\forall m = 0, 1, 2, ...$ we have $\sum_{\tau'=1}^{T-1}\mathbb{E}\{q_k(mT+\tau')\} \leq \mathbb{E}\{q_k(mT)\} + \frac{(T-1)T}{2}A_{max}$. Replacing we get

$$\begin{aligned}\lim_{t\to+\infty}\sup\frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\{q_k(\tau)\} &\leq \lim_{M\to\infty}\sup\left(\frac{2}{MT}\sum_{m=0}^{M-1}\mathbb{E}\{q_k(mT)\} + \frac{T(T-1)}{2}A_{max}\right)\\ &\leq \frac{2C_0}{T} + \frac{T(T-1)}{2}A_{max} < \infty,\end{aligned}$$

which implies that $\mathbf{q}(t)$ is stable. □

The above Lemma implies that a throughput optimal policy for the process $\tilde{\mathbf{q}}(m)$ should be also throughput optimal for the original system.

Define now $V(\mathbf{x}) = \frac{1}{2}\sum_{k=1}^{K}x_k^2$ a Lyapunov function and $\Delta V(\mathbf{x})$ its drift, i.e.

$$\Delta V(\mathbf{x}) = \mathbb{E}\left\{V(\tilde{\mathbf{q}}(m+1)) - V(\tilde{\mathbf{q}}(m))|\tilde{\mathbf{q}}(m) = \mathbf{x}\right\}. \tag{4.23}$$

The expectation is over the arrival and channel processes as well as the possibly randomized feedback policy (i.e. the set of F users that feed back). Then the following holds for the drift of the sampled system:

Lemma 4.4.2. *The drift of the quadratic Lyapunov function for the system $\tilde{\mathbf{q}}(m)$ under a scheduling policy π is upper bounded as follows (note that the* *number of users to feed back, F is included in the policy):*

$$\Delta V_{\pi}(\tilde{\mathbf{q}}(m)) \leq \tilde{B} + T\sum_{k=1}^{K}\tilde{q}_k(m)\lambda_k - (T-1)\sum_{k=1}^{K}\tilde{q}_k(m)\mathbb{E}\left\{\tilde{\mu}_k^{\pi}(m)|\tilde{\mathbf{q}}(m)\right\} \quad (4.24)$$

where $\tilde{\mu}_k^{\pi}(m)$ is the rate of user k in any of the slots $mT+1, ..., mT+(T-1)$ for a given channel state realization and outcome of the policy π (i.e. set of users actually fed back), $\tilde{B}$ is a constant depending only on the system parameters. The expectation is taken over the joint distribution of the channels and possible randomization of the policy.

Proof. Please refer to Section 4.11.2 in the Appendix of this Chapter for the proof. □

As a final remark, we note that the same stability results with Lemma 4.4.1 hold for the system operating under the centralized policy as well. That is, the system operating under the centralized policy is stable if and only if the system that results from sampling the queue lengths of the original at timeslots mT is strongly stable;the proof is essentially the same as the proof of Lemma 4.4.1.

4.5 A Special Case: The 2-User MISO BC With Single Rate

In this Section we will consider a simple case, namely a system with $K = 2$ users with identical channel statistics (i.i.d. Rayleigh with mean power gain $\bar{g}$) where a user gets rate R bits per channel use if SNR exceeds the threshold $\hat{S}$ and zero otherwise. This setting is of interest as the stability regions admit easy mathematical expressions and can be plotted, thus giving some insight on the outcomes of the policies.

To begin with, we define some parameters to be used frequently in the sequel. Define the probabilities that a user's SNR exceeds the threshold if only one or both users are scheduled as $\bar{p}(1)$ and $\bar{p}(2)$, respectively. Since the channels are statistically identical for both users, these probabilities are the same for any of them. The numerical values of these probabilities are given by

$$\begin{aligned}\bar{p}(1) &= 1 - \frac{\gamma\left(\frac{\hat{S}}{\bar{g}}\right); N}{\Gamma\left(N\right)} \\ \bar{p}(2) &= 1 - \frac{\gamma\left(\frac{2\sigma^2}{\bar{g}P}\hat{S}; N-F+1\right)}{\Gamma(N-F+1)}.\end{aligned} \quad (4.25)$$

These expressions are derived by specializing the results in Sections 4.2 and 4.4 for $F = 2$ and $\bar{g}_k = \bar{g}$. The system parameters are the same as in the original description: Downlink training requires β_p channel uses and uplink training requires pilots of length β channel uses for each user.

We now turn to characterizing the form and stability region of each policy.

4.5.1 Centralized policy

In this policy, in every slot t the transmitter selects either one or both the receivers to be scheduled. In the latter case, there is an overhead of $2\beta_c$ channel uses to broadcast the IDs of the two users and in the former, of $\beta_c + 1$ to broadcast the ID of the scheduled user and a signal that the control period is over.

The expected rate that a user gets if both users are scheduled or if this user only is scheduled at timeslot t is given by

$$\bar{\mu}_c(2) = (T_s - (\beta_p + 2\beta_c + 2\beta))\bar{p}(2)R \tag{4.26}$$

and by

$$\bar{\mu}_c(1) = (T_s - (1 + \beta_p + \beta_c + \beta))\bar{p}(1)R, \tag{4.27}$$

respectively. The set to be scheduled at slot t is then chosen at the beginning of this slot by the rule that follows:

$$\mathcal{F}_c(t) = \begin{cases} \{1,2\}, \text{ if } (q_1(t) + q_2(t))\bar{\mu}_c(2) \geq \max\{q_1(t), q_2(t)\}\bar{\mu}_c(1) \\ \mathcal{F}_c(t) = \{\arg\max\{q_1(t), q_2(t)\}\}, \text{otherwise} \end{cases}$$

The stability region of the system under this policy is characterized as follows:

Theorem 4.5.1. *The stability region of the centralized policy in the 2 i.i.d. user case with one rate level is*

$$\Lambda_c^{(2)} = \mathcal{CH}\Big\{(0, \bar{\mu}_c(1)), (\bar{\mu}_c(2), \bar{\mu}_c(2)), (\bar{\mu}_c(1), 0)\Big\}. \tag{4.28}$$

Proof. First we will prove that the region in the statement of the Theorem is indeed achievable by the centralized policy. To this end, consider a randomized policy that at the beginning of timeslot t selects the set $\mathcal{F}.$ of users to serve with probability $\pi_{\mathcal{F}}$. In our setting the set $\mathcal{F}$ can be one of $\{1\}, \{2\}, \{1,2\}$. The achievable stability region by this policy is

$$\begin{array}{c} \lambda_1 < \pi_{\{1\}}\bar{\mu}_c(1) + \pi_{\{1,2\}}\bar{\mu}_c(2) := \hat{\mu}_1 \\ \lambda_1 < \pi_{\{2\}}\bar{\mu}_c(1) + \pi_{\{1,2\}}\bar{\mu}_c(2) := \hat{\mu}_2 \end{array}, \tag{4.29}$$

with $\pi_{\{1\}} + \pi_{\{2\}} + \pi_{\{1,2\}} \leq 1$ and $0 \leq \pi_{\mathcal{F}} \leq 1$. This is exactly the algebraic characterization of the set (4.28).

Define now the quadratic Lyapunov function $V(\mathbf{x}) = x_1^2 + x_2^2$. Its drift $\Delta V(\mathbf{q}) = \mathbb{E}\left\{V(\mathbf{q}(t+1)) - V(\mathbf{q}(t))|\mathbf{q}(t) = \mathbf{q}\right\}$ can be shown to be bounded as (with some positive constant B)

$$\Delta V(\mathbf{q}) \leq B - \sum_{k=1}^{2}(q_k(t)\mathbb{E}\{\mu_k(t)\} - q_k(t)\lambda_k) \leq B - \sum_{k=1}^{2} q_k(t)(\hat{\mu}_k - \lambda_k).$$

The second inequality follows by the definition of our centralized policy. From (4.29) it follows that $\forall \lambda \in \Lambda_c^{(2)}, \exists \epsilon > 0$ such that $\Delta V(\mathbf{q}) \leq B - \epsilon \sum_{k=1}^{2} q_k(t)$, hence the system under the centralized policy is indeed stable for all mean arrival rates in the asserted region.

We then need to prove the converse, that is, if a centralized policy achieves stability, then the mean arrival rate lies in (the interior of) the region given by (4.28). Indeed, assume that the system is stable for a mean arrival rate vector λ. The centralized policy depends only on the queue lengths at the beginning of slot t, which we denote by $\mathcal{F}(\mathbf{q})$. The assumptions, thus, on the channel and arrival processes make the system a discrete time Markov chain with a single communicating class. In this case, stability implies the existence of an invariant distribution $\pi(\mathbf{q})$. The mean service rate user k gets is then equal to

$$\lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mu_k(\tau) = \bar{\mu}_c(1) \sum_{\mathbf{q}\in\mathbb{Z}_+^2:\mathcal{F}(\mathbf{q})=\{k\}} \pi(\mathbf{q}) + \bar{\mu}_c(2) \sum_{\mathbf{q}\in\mathbb{Z}_+^2:\mathcal{F}(\mathbf{q})=\{1,2\}} \pi(\mathbf{q}).$$

Since the sums are probabilities themselves, we can see that the service rate have the same form as in (4.29). Also, since the system is assumed stable, there should be $\lambda_k < \lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mu_k(\tau)$. From these we conclude that $\lambda \in \Lambda_c^{(2)}$, completing the proof. $\square$

The stability region for the centralized scheduling algorithm looks like a trapeze with corner points $(0,0), (0, \bar{\mu}_c(1)), (\bar{\mu}_c(2), \bar{\mu}_c(2)), (\bar{\mu}_c(1), 0)$ if it holds that $\bar{\mu}_c(1) < 2\bar{\mu}_c(2)$ and like a triangle with corners at $(0,0), (0, \bar{\mu}_c(1)), (\bar{\mu}_c(1), 0)$ otherwise. This follows from the fact that in the latter case only one user will get scheduled. The region for the former case is illustrated in Fig. 4.4.

4.5.2 Decentralized Policy

Let the time for contention be τ_c channel uses. For consistency, the contention period will be present even if $F = 2$, i.e. when both users are to be scheduled (this will be improved by the mixed policy). From the results of Section 4.4.2 it suffices to look at the stability of the system examined in the beginning of each signalling slot $mT, m = 0, 1, ...$

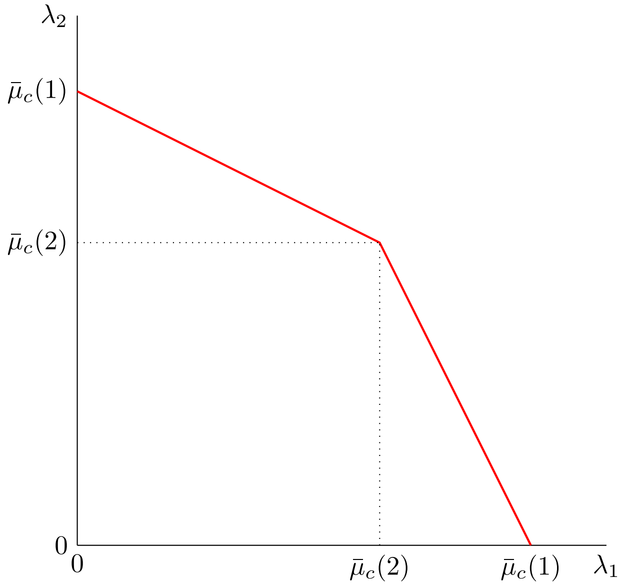

Figure 4.4: Stability region of the centralized policy for a system with 2 users with i.i.d. channels and a single rate level

As described in Section 4.3.2, the BS broadcasts the quantized queue lengths $\hat{q}_1(mT), \hat{q}_2(mT)$ at time mT. In the particular case with 2 users, the decision taken by the BS is to either select both users ($F(m) = 2$) or signal that one user will be selected ($F(m) = 1$) and the user who will be scheduled is the one with the lowest timer from eq. (4.9).

1) Contention procedure: If $F(m) = 1$, in each of these slots the receivers are given a contention period of τ_c channel uses to decide which one is to be scheduled based on the (quantized and outdated) queue length information they have and the realization of their channels. This can be done using a contention scheme, assuming contention in continuous time e.g. like [52], where each user waits until time $\frac{\tau_c}{\hat{q}_k(m)r_k(t)}$: if both have the same timer, e.g. the user with the smallest ID is scheduled. Another alternative, that can be used thanks to our model, is to divide the contention period into minislots (TDMA manner) where each receiver sends a signal in its corresponding minislot if its SNR is above the threshold $\hat{S}$. If both receivers send a signal, in their corresponding minislots, then the receiver with the largest broadcasted queue length gets scheduled for training (this analysis/comparison can be done independently by each receiver since the queue lengths of all receivers are broadcasted). Otherwise, if only one user sends a signal in a minislot, then this user will be scheduled for training. Then, the user to be scheduled sends its ID to the BS , taking β_c channel uses, and trains. Using the above "decentralized" procedure, the user that will eventually get served in the slot will be the one with the maximum product of quantized queue length at mT times achievable rate. Due to our model here, denoting $SNR_k^{(1)}(t) = \frac{Pg_k(t)}{\sigma^2}$, the user to be scheduled will be

- If $\forall k = 1, 2$ holds $SNR_k^{(1)}(t) > \hat{S}$, then $k^*(t) = \arg\max[\hat{q}_1(mT), \hat{q}_2(mT)]$
- The user for which $SNR_k^{(1)}(t) > \hat{S}$ otherwise

The scheduled receiver will always be given rate of R bits per channel use, except in the case where no one has sufficiently high SNR, in which no receiver can be scheduled anyway. Defining the permutation $k(1), k(2)$, where $\hat{q}_{k(1)}(mT) \geq \hat{q}_{k(2)}(mT)$, the average service rates of these users under $F = 1$ for the next $T - 1$ slots are

$$\begin{aligned} \bar{\mu}_{k(1)}^{d,(1)}(t) &= (T_s - (\beta_p + \tau_c + \beta))\bar{p}(1)R := \bar{\mu}_d(1) \\ \bar{\mu}_{k(2)}^{d,(1)}(t) &= (T_s - (\beta_p + \tau_c + \beta))\bar{p}(1)(1 - \bar{p}(1))R. \end{aligned} \tag{4.30}$$

2) F(m)=2: Both users train just after the coordination period. The average rate per slot for each user in this case will be

$$\bar{\mu}_d(2) = (T_s - (\beta_p + \tau_c + 2\beta))\bar{p}(2)R \tag{4.31}$$

Based on the above, the transmitter decides at $t = mT$ the number of users to get scheduled for the next $T - 1$ slots by:

$$F(m) = \begin{cases} 2, \text{if } \hat{q}_{k(1)}(m) + \hat{q}_{k(2)}(m))\bar{\mu}_{k(1)}^{d,(1)}(t) \geq \\ (\hat{q}_{k(1)}(m) + \hat{q}_{k(2)}(t)(1 - \bar{p}(1)))\bar{\mu}_{k(1)}^{d,(1)}(t) \\ 1, \text{otherwise} \end{cases}$$

In the case of $F(m) = 1$, the contention procedure is followed.

The stability region of this policy is described as follows :

Theorem 4.5.2. *The stability region of the decentralized scheme for the* 2 *user MISO broadcast system with a single rate level is*

$$\Lambda_d^{(2)} = \left(1 - \frac{1}{T}\right)\mathcal{CH}\Big\{(0, \bar{\mu}_d(1)), (\bar{\mu}_d(1)(1 - \bar{p}(1)), \bar{\mu}_d(1)), \\ (\bar{\mu}_d(2), \bar{\mu}_d(2)), (\bar{\mu}_d(1), \bar{\mu}_d(1)(1 - \bar{p}(1))), (\bar{\mu}_d(1), 0)\Big\}$$

Proof. The proof consists in four parts. For the first two parts we compute the stability region for policies that select all the time $F = 2$ and $F = 1$. Then we prove the convex combination of the two is achievable by the decentralized policy and we finish by proving the converse. In the proof we examine the system $\tilde{\mathbf{q}}(m) = \mathbf{q}(mT)$, since from Lemma 4.4.1 stability of this system is sufficient for stability of the original queueing system.

Step 1: We first find the stability region if $F = 2$ for every signalling slot mT. In this case, the mean rate a user gets for each data slot is $\bar{\mu}_d(2)$. Thus, for the system $\tilde{\mathbf{q}}(m)$, the mean arrival rate for user k is $T\lambda_k$ and the mean service rate is $(T-1)\bar{\mu}_d(2)$, thus the stability region here is $\lambda_k < \frac{T-1}{T}\bar{\mu}_d(2), \forall k = 1, 2$.

Step 2: We then find the stability region if $F = 1$ in every signalling slot. We define a hypothetical policy where a the BS knows from the start of a data slot the achievable rates for both users and, based on this knowledge, chooses one of the two users to train and get scheduled, probably at random (while keeping the same time for data transmission in the slot as the corresponding in the decentralized policy). More concretely, only one user can support the rate R then this user should be scheduled, otherwise if both support the rate R then user 1 gets scheduled with some probability π_1 and user 2 with a probability π_2. In this case, taking into account the model for the system $\tilde{\mathbf{q}}(m)$ the mean arrival rates λ_1, λ_2 that can be supported by the system are the one for which there exist probabilities π_1, π_2 such that (the quantities in the right hand side are the mean rates given to each user):

$$\begin{aligned} T\lambda_1 &< (T-1)\left((1-\bar{p}(1))\bar{\mu}_d(1) + \pi_1\bar{p}(1)\bar{\mu}_d(1)\right) := (T-1)\hat{\mu}_{d,1} \\ T\lambda_2 &< (T-1)\left((1-\bar{p}(1))\bar{\mu}_d(1) + \pi_2\bar{p}(1)\bar{\mu}_d(1)\right) := (T-1)\hat{\mu}_{d,2} \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 0 \leq \pi_1 + \pi_2 \leq 1. \end{aligned} \tag{4.32}$$

This is (for λ) the algebraic representation of the convex hull of the points $(0, \frac{T-1}{T}\bar{\mu}_d(1))$, $(\frac{T-1}{T}(1-\bar{p}(1))\bar{\mu}_d(1), \frac{T-1}{T}\bar{\mu}_d(1))$, $(\frac{T-1}{T}\bar{\mu}_d(1), \frac{T-1}{T}(1-\bar{p}(1))\bar{\mu}_d(1))$, $(\frac{T-1}{T}\bar{\mu}_d(1), 0)$. Now assume a vector λ inside this region denoting $\hat{\mu}_k$ the mean rate of user k under a hypothetical policy such that the system is stable. From Lemma 4.4.2 we have that the drift of the quadratic Lyapunov function here is

$$\Delta V_\pi(\tilde{\mathbf{q}}(m)) \leq \tilde{B} + T\sum_{k=1}^{K}\tilde{q}_k(m)\lambda_k - (T-1)\sum_{k=1}^{K}\tilde{q}_k(m)\mathbb{E}\left\{\tilde{\mu}_k^\pi(m)|\tilde{\mathbf{q}}(m)\right\}$$

Recall that $\hat{\mathbf{q}}(m)$ is vector containing the quantized versions of the queue lengths at the beginning of the signalling slot, therefore $\tilde{q}_k(m) - Q \leq \hat{q}_k(m) \leq \tilde{q}_k(m) + Q$. Also that the decentralized policy here selects the user with the maximum product of rate times quantized queue length, thus we get

$$\begin{aligned}
\Delta V_{\pi}(\tilde{\mathbf{q}}(m)) &\leq \tilde{B} + T\sum_{k=1}^{K}\tilde{q}_k(m)\lambda_k - (T-1)\sum_{k=1}^{K}\tilde{q}_k(m)\mathbb{E}\left\{\tilde{\mu}_k^d(m)|\tilde{\mathbf{q}}(m)\right\} \\
&\leq \tilde{B} + TQ\sum_{k=1}^{2}\lambda_k + (T-1)KR_{max}Q + T\sum_{k=1}^{K}\hat{q}_k(m)\lambda_k \\
&- (T-1)\sum_{k=1}^{K}\hat{q}_k(m)\mathbb{E}\left\{\tilde{\mu}_k^d(m)|\tilde{\mathbf{q}}(m)\right\} \\
&\leq \tilde{C} + \sum_{k=1}^{2}\hat{q}_k(m)\left(\lambda_k - \hat{\mu}_k\right) \\
&\leq \tilde{C} - \epsilon\sum_{k=1}^{2}\hat{q}_{d,k}(m).
\end{aligned}$$

The drift is negative for $\sum_{k=1}^{2}\hat{q}_k(m) > \tilde{C}/\epsilon \implies \sum_{k=1}^{2}\hat{q}_k(m) > 2Q + \tilde{C}/\epsilon$, thus the system under the decentralized policy achieves indeed the stability region given by (4.32).

Step 3: Here we prove that $\Lambda_d^{(2)}$ is achievable by the decentralized policy. Consider a randomized policy between $F = 1$ and $F = 2$ with probabilities $\pi(F = 1)$ and $\pi(F = 2)$ (independent on anything), respectively and the randomized hypothetical policy for the case of $F = 1$ given in the above paragraph. The mean arrival rates supported under this policy should then be such that there exist these probabilities while satisfying the conditions

$$\begin{aligned}
&T\lambda_k < (T-1)\left(\pi(F=1)\left((1-\bar{p}(1))\bar{\mu}_d(1) + \pi_k\bar{p}(1)\bar{\mu}_d(1)\right) + \pi(F=2)\bar{\mu}_d(2)\right) \\
&:= (T-1)\hat{\mu}_{d,k}, k = 1, 2 \\
&0 \leq \pi(F=1) + \pi(F=2) \leq 1 \\
&0 \leq \pi_1 + \pi_2 \leq 1.
\end{aligned} \tag{4.33}$$

The region defined by the above equations is the convex hull of the two regions defined by (4.32) and (4.30), thus the set in the statement of the theorem. Under the proposed policy, using the same calculations as above, the drift of the quadratic Lyapunov function becomes

$$\begin{aligned}
\Delta V_{\pi}(\tilde{\mathbf{q}}(m)) &\leq \tilde{C} + T\sum_{k=1}^{2}\hat{q}_k(m)\lambda_k - (T-1)\sum_{k=1}^{2}\hat{q}_k(m)\mathbb{E}\{\mu_k^{\pi}(m)\} \\
&\leq \tilde{C} + \sum_{k=1}^{2}\hat{q}_k(m)(T\lambda_k - (T-1)\hat{\mu}_{d,k}),
\end{aligned}$$

where the second inequality follows from the fact that by definition of the policy the quantity $\sum_{k=1}^{2}\hat{q}_k(m)\mathbb{E}\{\mu_k^{\pi}(m)\}$ is maximized. Then, by (4.33) we get that for some $\epsilon > 0$, $\Delta V_\pi(\tilde{\mathbf{q}}(m)) < \tilde{C} - \epsilon \sum_{k=1}^{2} \hat{q}_k(m)$, which is negative for (as above) $\sum_{k=1}^{2} \tilde{q}_k(m) \geq 2Q + \frac{\tilde{C}}{\epsilon}$, therefore the decentralized policy can support any rate of the (interior of the) set in the statement of the Theorem.

Step 4: To finish, we prove the converse, that is any mean arrival rate vector λ for which the system under the decentralized policy is stable lies in the interior of the set $\Lambda_d^{(2)}$. We have that (i) the number of users scheduled by the decentralized policy depends on the quantized queue lengths and that the user scheduled for $F = 1$ depends on the quantized queue length and the channel state realizations and (ii) the quantized queue lengths are functions of the actual queue lengths at the start of slot mT, the system is an aperiodic markov chain with countable state space ($\mathbb{Z}_+^2$) and a single communicating class, thus strong stability implies ergodicity of the chain, therefore existence of an invariant distribution $\pi(\mathbf{q})$. The mean service rate a user 1 gets is therefore

$$\lim_{M\to\infty} \frac{1}{M} \sum_{m=0}^{M-1} \sum_{t=mT+1}^{mT+T-1} \mu_1(t) = (T-1)\Bigg(\bar{\mu}_d(1) \sum_{\mathbf{q}\in\mathbb{Z}_+^2 : F(\mathbf{q})=1, \hat{q}_1 \geq \hat{q}_2} \pi(\mathbf{q})$$

$$+ (1-\bar{p}(1))\bar{\mu}_d(1) \sum_{\mathbf{q}\in\mathbb{Z}_+^2 : F(\mathbf{q})=1, \hat{q}_1 < \hat{q}_2} \pi(\mathbf{q}) + \bar{\mu}_d(2) \sum_{\mathbf{q}\in\mathbb{Z}_+^2 : F(\mathbf{q})=2} \pi(\mathbf{q})\Bigg)$$

and similar for user 2. By assumption the system is stable therefore $T\lambda_k < \lim_{M\to\infty} \frac{1}{M} \sum_{m=0}^{M-1} \sum_{t=mT+1}^{mT+T-1} \mu_k(t)$ for both users. Combining the above, and since the summations in the right hand side of the mean rate expression are probabilities, we get that $\lambda \in \Lambda_d^{(2)}$. □

Illustration of the stability region achieved by the decentralized policy is shown in Fig. 4.5.

In the special case of $K = 2$ we consider here, if the overhead for the contention τ_c is 2 channel uses then the scheme can be implemented even dropping the assumption of continuous time for contention. Indeed, the first of the two channel uses can be dedicated for the user with the biggest quantized queue length and the second for the other user (since the queue lengths are broadcasted, each users knows the queue length of the other), with the ranking be based on the user ID in case of a tie. Then, for $F = 1$, the first user in the ranking sends a signal if its SNR exceeds the threshold and remains silent otherwise, same for the second user. Note that based on the same idea, we can have even $\tau_c = 1$ channel use: the first user in the ranking only signals if its channel can support the rate, if yes the user is scheduled, if not the other user is scheduled (though this would consume extra power from the BS if the channel of other user is also bad).

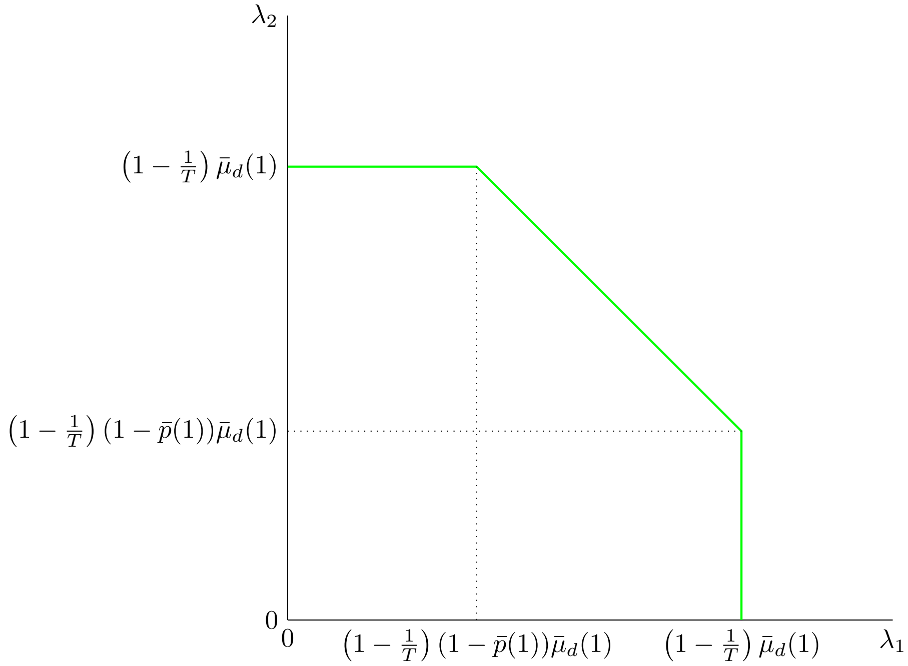

Figure 4.5: Stability region of the decentralized policy for a system with 2 users with i.i.d. channels and a single rate level.

4.5.3 Mixed Policy

The mixed policy is a combination of both the ideas behind the centralized and decentralized policies. As in the decentralized policy, slot mT is used to broadcast signalling regarding the quantized queue lengths and the action that specifies how scheduling will be done in the next $T-1$ slots.

In the signalling slot, the BS there can choose one of the following actions: $\mathcal{F}=\{1\}$, $\mathcal{F}=\{2\}$, $\mathcal{F}=\{1,2\}$ and $F=1$. In the first three actions the user(s) specified train directly in the uplink for the $T-1$ slots after the signalling slot, without any control or contention/uplink of the IDs phase. In the case of $F=1$ one user is scheduled according to the contention procedure explained in Section 4.5.2. In detail, for the rates at a slot t corresponding to each of the BS actions and assuming $\hat{q}_{k(1)}(m) \geq \hat{q}_{k(2)}(m)$ we have for $t \in \{mT+1, ..., mT+T-1\}$:

$$\begin{aligned}
&\mathbb{E}\left\{\mu_1(t)\right\} = (T_s - (\beta_p + \beta))\bar{p}(1)R, \mu_2(t) = 0, \mathcal{F} = 1 \\
&\mathbb{E}\left\{\mu_1(t)\right\} = 0, \mu_2(t) = (T_s - (\beta_p + \beta))\bar{p}(1)R, \mathcal{F} = 2 \\
&\mathbb{E}\left\{\mu_1(t)\right\} = \mathbb{E}\left\{\mu_2(t)\right\} = (T_s - (\beta_p + 2\beta))\bar{p}(1)R, \mathcal{F} = \{1,2\} \\
&\mathbb{E}\left\{\mu_{k(1)}(t)\right\} = \bar{\mu}_d(1) \\
&\mathbb{E}\left\{\mu_{k(2)}(t)\right\} = (1 - \bar{p}(1))\bar{\mu}_d(1), F = 1.
\end{aligned} \tag{4.34}$$

We define further $\bar{\mu}_m(\{k\}) = (T_s - (\beta_p + \beta))\bar{p}(1)R$ and $\bar{\mu}_m(\{1,2\}) = (T_s - (\beta_p + 2\beta))\bar{p}(2)R$. The mixed policy selects, at every slot mT, the following action:

- $\mathcal{F} = \{k(1)\}$, if
  $\hat{q}_k(1)(mT)\bar{\mu}_m(\{k\}) >$
  $\max\left\{(\hat{q}_1(mT) + \hat{q}_2(mT))\bar{\mu}_m(\{1,2\}), (\hat{q}_1(mT) + (1-\bar{p}(1))\hat{q}_2(mT))\bar{\mu}_d(1)\right\}$
- $\mathcal{F} = \{1,2\}$, if
  $(\hat{q}_1(mT) + \hat{q}_2(mT))\bar{\mu}_m(\{1,2\}) \leq$
  $\max\left\{\hat{q}_k(1)(mT)\bar{\mu}_m(\{k\}), (\hat{q}_1(mT) + (1-\bar{p}(1))\hat{q}_2(mT))\bar{\mu}_d(1)\right\}$
- $F = 1$ if
  $(\hat{q}_1(mT) + (1-\bar{p}(1))\hat{q}_2(mT))\bar{\mu}_d(1) >$
  $\max\left\{\hat{q}_k(1)(mT)\bar{\mu}_m(\{k\}), (\hat{q}_1(mT) + \hat{q}_2(mT))\bar{\mu}_m(\{1,2\})\right\}$

The main result here is summarized in the following

Theorem 4.5.3. *The stability region of the mixed scheme in the* 2 *user case with i.i.d. channels and one rate level is*

$$\begin{aligned}
\Lambda_m^{(2)} = \left(1 - \frac{1}{T}\right)\mathcal{CH}\Big\{&(0, \bar{\mu}_m(\{k\})), (\bar{\mu}_d(1), (1-\bar{p}(1))\bar{\mu}_d(1)), (\bar{\mu}_m(\{1,2\}), \bar{\mu}_m(\{1,2\})), \\
&((1-\bar{p}(1))\bar{\mu}_d(1), \bar{\mu}_d(1)), (\bar{\mu}_m(\{k\}), 0)\Big\}.
\end{aligned} \tag{4.35}$$

Proof. The proof is in the same spirit as the proof of Theorem 4.5.2, that is deriving the stability region of every action first. Due to the high similarity for the proofs of Theorems 4.5.1 and 4.5.2, only the outline is given to avoid repetition.

The stability region for the action $F = 1$ for every signalling slot has already been derived in the proof of Theorem 4.5.2. In addition, if the action $\mathcal{F} = \{1,2\}$ is chosen all the time, the mean arrival rates that can be supported must satisfy

$$T\lambda_k < (T-1)\bar{\mu}_m(\{1,2\}), \forall k \in \{1,2\}.$$

Finally, if only the user with the biggest (quantized) queue length at the beginning of slot mT is scheduled in the slots $t \in \{mT+1, .., mT+T-1\}$, the region with mean arrival rates such that there exist probabilities $\pi_{\{1\}}, \pi_{\{2\}}$ so that

$$\begin{aligned} T\lambda_1 &< (T-1)\pi_{\{1\}}\bar{\mu}_m(\{k\}) \\ T\lambda_2 &< (T-1)\pi_{\{2\}}\bar{\mu}_m(\{k\}) \\ 0 \leq \pi_{\{1\}} &+ \pi_{\{2\}} \leq 1 \end{aligned} \tag{4.36}$$

is satisfied. The proof uses the same ideas with Theorem 4.5.1 (i.e. the randomized policy) and the bound of the Lyapunov drift including the quantized queue lengths seen in theorem Theorem 4.5.2. Same arguments as the ones of Theorem 4.5.1 give that the mixed policy achieves the stability region of this Theorem and the converse, i.e. that every mean arrival rate vector for which the mixed policy stabilizes the system is in the stability region given in the Theorem. □

In short, as with the centralized and decentralized policies, the mixed policy aims to maximize the quantity $\mathbb{E}\left\{\sum_{k=1]}^{2} \hat{q}_k(mT)\mu_k(t)|\hat{\mathbf{q}}(mT)\right\}$ over all allowed actions. An illustration of the stability region achieved by the mixed policy is given in Fig. 4.6.

4.5.4 Comparison and Discussion

In all three cases the main idea of the policies is to try to maximize the negative part of the drift of the quadratic Lyapunov function. The difference between them lies in the fact that different constraints on the policy are assumed. In the centralized policy the BS is based on the queue lengths on the beginning of each slot. This has the benefit of knowing the "priority" a user has to get scheduled in real time but the drawback that it can lead to many rate outages in slots. By broadcasting the queue lengths (even in quantized versions) periodically and by being based on the queue lengths every slot mT it is more possible that some rate may be wasted, i.e. a user may be scheduled even if the queue length at the current slot is empty. In addition, for the decentralized case, since one every T slots must be used for signalling and some overhead for contention and uplink of the ID of the user who won the contention should be used in each data slot, there is some penalty on the overall rate. On the other hand, letting the users decide according to their instantaneous channel states leads to scheduling eventually a user with good channel condition (in the case where the BS selects $F = 1$).

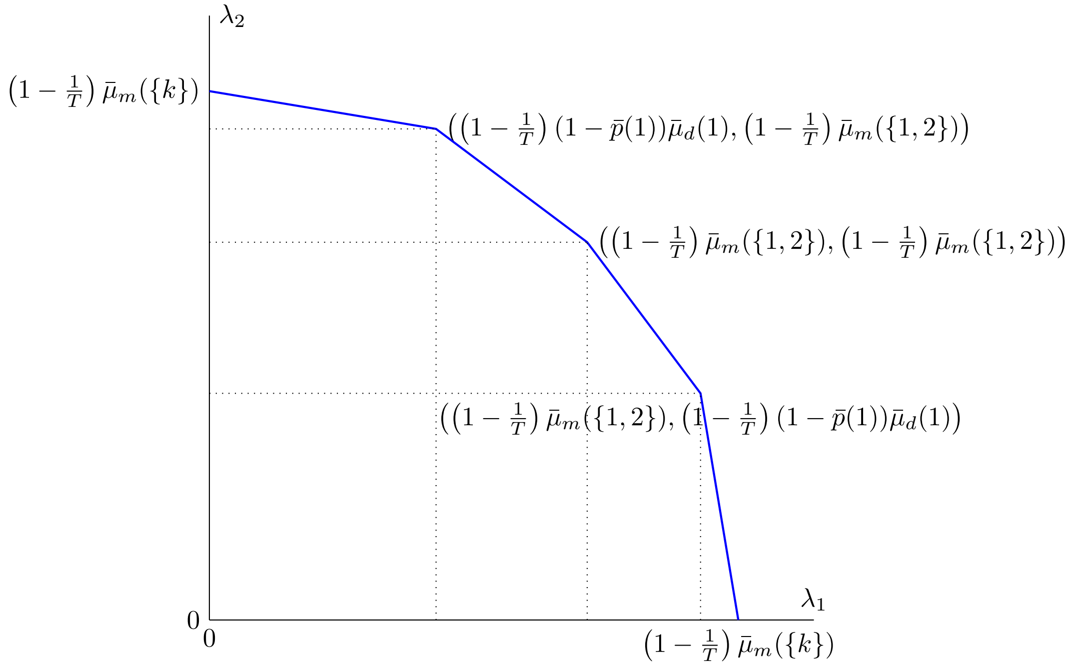

Figure 4.6: Stability region of the mixed policy for a system with 2 users with i.i.d. channels and single rate level.

Proposition 4.5.1. *A sufficient condition for the mixed policy to achieves a bigger stability region than the centralized policy is*

$$T > \max\left[\frac{T_c-\beta_p-\beta}{1+\beta_c}, \frac{T_c-\beta_p-2\beta}{2\beta_c}\right]. \tag{4.37}$$

In addition, this increase is with a factor at least equal to

$$\rho(T) = \frac{T}{T-1}\min\left[\frac{T_c-\beta_p-\beta}{T_c-(\beta_p+\beta_c+1+\beta)}, \frac{T_c-\beta_p-2\beta}{T_c-(\beta_p+2\beta_c+2\beta)}\right] \tag{4.38}$$

that is $\Lambda_m^{(2)} \supseteq \rho(T)\Lambda_c^{(2)}$.

Proof. We will take the points on the axes and on the line $\lambda_1 = \lambda_2$; from the shapes of the stability regions, if the mixed policy expands the region along these directions, then it will expand it anywhere. For expansion, we should then have

$$\left(1-\frac{1}{T}\right)\bar{\mu}_m(\{k\}) > \bar{\mu}_c(1)$$

and

$$\left(1-\frac{1}{T}\right)\bar{\mu}_m(\{1,2\}) > \bar{\mu}_c(2).$$

Replacing and after come calculations we get that

$$T > \frac{T_c - \beta_p - \beta}{1 + \beta_c}$$

from the first equation and

$$T > \frac{T_c - \beta_p - 2\beta}{2\beta_c}$$

from the second. Since both must be correct we get the stated condition for T. In addition, from the shape of the regions we get that for all other directions we get a bigger increase than the increase on the axes and the like $\lambda_1 = \lambda_2$. The increase in any of the two axes is given as $\rho' = \frac{(1-1/T)\bar{\mu}_m(\{k\})}{\bar{\mu}_c(1)}$ and the increase in the direction of equal arrival rates is $\rho'' = \frac{(1-1/T)\bar{\mu}_m(\{1,2\})}{\bar{\mu}_c(2)}$. Replacing we get the stated result. □

We can note that the proof of the above proposition uses only points achieved by the periodic centralized policy. That is, the increase comes from the fact that a smaller overhead for training and signalling in the data slots is needed and the necessary overhead for scheduling is in the slots mT instead. The use of decentralized scheme in the mixed policy helps enlarge the stability region outside the lines connecting the point $((1-1/T)\bar{\mu}_m(\{1,2\}), (1-1/T)\bar{\mu}_m(\{1,2\}))$ with the points on the axes thus yielding more gains with respect to the centralized region for traffic demands in these directions[6]. Illustration of both regions and the comparison is given in Fig. 4.7.

As $T \to \infty$, this increase is bounded and the bound depends only on the parameters of the system (i.e. total channel uses available in one timeslot, lengths of pilot sequences and rates for control signals); this limit is the highest stability region the mixed scheme can achieve. Finally, note that increasing T leads to a bigger stability region, however it may also lead to bigger delays and slower convergence of the system to its stationary behaviour.

4.6 General case

In this Section we consider the general case with K users and L possible transmission rates, as described in Section 4.2.

[6] If the overhead for contention and user ID uplink of the decentralized policy is smaller than the control overhead of the centralized one, then a result similar to Proposition 4.5.1 holds even if only the decentralized policy is used; however this may not be the case in general.

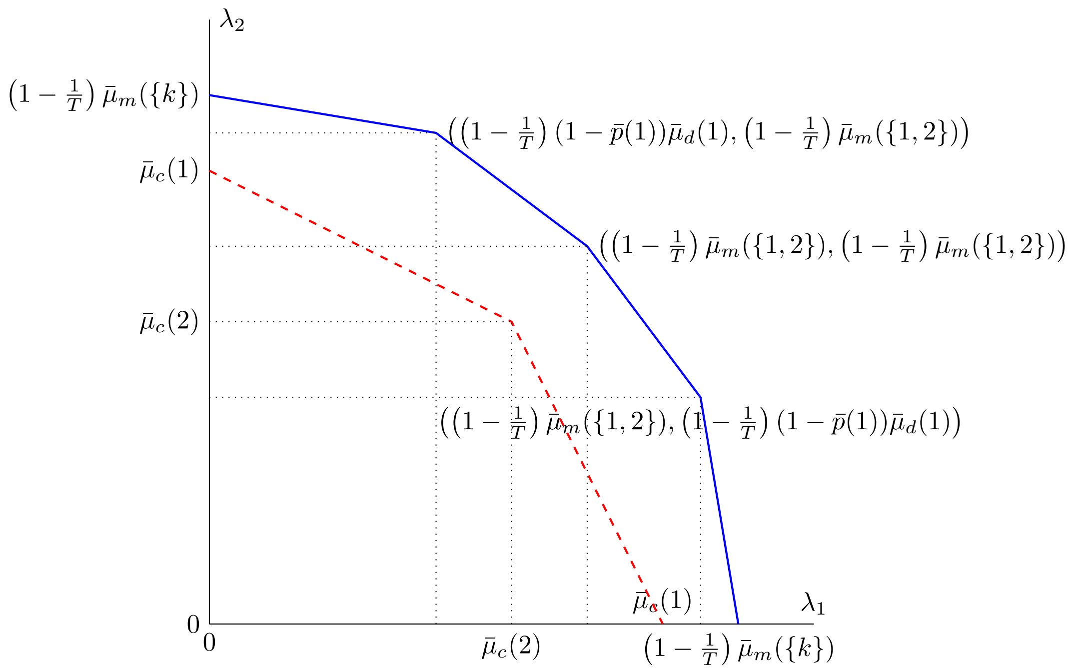

Figure 4.7: Comparing the stability regions of the centralized (dashed line) and mixed (continuous line) policies in the 2 user system with single rate level

4.6.1 Centralized Policy

We begin by considering the centralized policy and characterizing its stability region. We denote by $\mu_k^{(c)}(t)$ the service in bits given to user k at slot t under the centralized policy.

Theorem 4.6.1. *The stability region Λ_c of the centralized policy consists in all rate vectors $\lambda = [\lambda_1, .., \lambda_K]$ for which there exist $0 \leq p(\mathcal{F}) \leq 1, \mathcal{F} \in 2^{\mathcal{K}}$ with $\sum_{\mathcal{F}\in 2^{\mathcal{K}}} p(\mathcal{F}) = 1$ such that*

$$\lambda_k < \sum_{\mathcal{F}\in 2^{\mathcal{K}}} p(\mathcal{F}) I_{\{k\in\mathcal{F}\}}(T_s - (1+\beta_p+(\beta+\beta_c)|\mathcal{F}|))\mathbb{E}\left\{r_k||\mathcal{F}|\right\}, \forall k \in \mathcal{K}. \quad (4.39)$$

Proof. First we prove that the centralized policy achieves the region characterized above. Note that this stability region is achieved by a randomized policy that every time slot schedules users in the set $\mathcal{F}$ randomly with probability $p(\mathcal{F})$ such that for every user (4.39) is satisfied. Denoting μ_k^* the mean rate user k gets under this policy, satisfaction of the aforementioned condition implies that for some $\epsilon > 0$ there is $\mu_k^* - \lambda_k \geq \epsilon$. Defining the quadratic Lyapunov function $V(\mathbf{x}) = \frac{1}{2}\sum_{k=1}^{K} x_k^2$, we have under the centralized policy (the expectation is over the arrival, channel and scheduling policies):

$$\begin{aligned}\Delta V(\mathbf{q}) &= \mathbb{E}\left\{V(\mathbf{q}(t+1)) - V(\mathbf{q}(t))|\mathbf{q}(t)=\mathbf{q}\right\}\\ &\le B + \sum_{k=1}^{K} q_k(t)\lambda_k - \sum_{k=1}^{K} q_k(t)\mathbb{E}\left\{\mu_k^{(c)}(t)|\mathbf{q}(t)=\mathbf{q}\right\}\\ &\le B + \sum_{k=1}^{K} q_k(t)(\lambda_k - \mu_k^*) \le B - \epsilon\sum_{k=1}^{K} q_k(t),\end{aligned}$$

which implies that the system under the centralized policy is stable. The second inequality holds because the centralized policy chooses $\mathcal{F}$ to maximize the second sum; under the randomized policy the expectation of the rate of user k in every slot is μ_k^*, since this schedule is chosen independent of everything.

Conversely, assume that the centralized policy renders the system stable for a mean arrival rate vector λ in the interior of Λ_c. Then, the system is a Markov chain and since it is strongly stable it has a unique invariant distribution $\pi(\mathbf{q})$. In addition, since it is stable, there must hold $\lambda_k < \lim_{t\to\infty}\frac{1}{t}\sum_{\tau=0}^{t-1}\mu_k^{(c)}(\tau)$. On the other hand, the scheduling decision depends on the queue length only, this dependency denoted below by $\mathcal{F}(\mathbf{q})$, therefore the mean service rate for user k under the centralized policy is given as

$$\begin{aligned}\lim_{t\to\infty}\frac{1}{t}\sum_{\tau=0}^{t-1}\mu_k^{(c)}(\tau) &= \sum_{\mathbf{q}\in\mathbb{Z}_+^K}\pi(\mathbf{q})\mathbb{E}\left\{\mu_k^{(c)}(t)|\mathcal{F}(\mathbf{q})\right\}\\ &= \sum_{\mathcal{F}}\mathbb{E}\left\{\mu_k^{(c)}(t)|\mathcal{F}\right\}\sum_{\mathbf{q}\in\mathbb{Z}_+^K:\mathcal{F}(\mathbf{q})=\mathcal{F}}\pi(\mathbf{q}) > \lambda_k.\end{aligned}$$

Replacing for the service, the above can be written indeed as (4.39) by setting $p(\mathcal{F}) = \sum_{\mathbf{q}\in\mathbb{Z}_+^K:\mathcal{F}(\mathbf{q})=\mathcal{F}}\pi(\mathbf{q})$. □

Geometrically the stability region of the centralized policy is the convex hull generated by the points $\left(\mathbb{E}\left\{\mu_1^{(c)}(t)|\mathcal{F}\right\},...,\mathbb{E}\left\{\mu_K^{(c)}(t)|\mathcal{F}\right\}\right)$ for every $\mathcal{F}$ subset of $\{1,...,K\}$. The expectation is over the channel state distributions.

4.6.2 Decentralized Policy

Denoting as $\Lambda_d(F)$ the stability region of the decentralized policy for a fixed number of users to feed back every timeslot, i.e. setting $F(m)=F,\forall m\ge 0$. In this case, from the description of the contention scheme, the users with the F maximum values of $\hat{q}_k(t)\mathbb{E}\left\{\mu_k(t)|g_k(t),F\right\}$ will eventually get scheduled in every data slot t, where the channel state realizations are such that the magnitudes are $g_k(t)$. This observation leads to the following:

Lemma 4.6.1. *The region $\Lambda_d(F)$ consists in all mean arrival rate vectors $\lambda \in \mathbb{R}_+^K$ for which there exist $\phi_{\mathcal{F}}(\mathbf{g}) \geq 0$ such that*

$$\begin{aligned}
&T\lambda_k < (T-1)\int_0^\infty p_1(g_1)dg_1 ... \int_0^\infty p_K(g_K)dg_K \sum_{\mathcal{F}:|\mathcal{F}|=F} \phi_{\mathcal{F}}(\mathbf{g})\mathbb{E}\left\{\mu_k(t)|g_k,\mathcal{F}\right\} \\
&:= (T-1)\mu_k^*(F) \qquad . \\
&\sum_{\mathcal{F}:|\mathcal{F}|=F} \phi_{\mathcal{F}}(\mathbf{g}) \leq 1, \forall \mathbf{g} \in \mathbb{R}_+^K
\end{aligned} \tag{4.40}$$

The expectation is with respect to the directions of the channel vectors and $\mathbf{g}$ *is the vector containing a realization of the channel magnitudes.*

Proof. The region above is achieved by a hypothetical policy where the BS at each slot t knows the realizations of all the channel magnitudes and, based on this knowledge, selects each set $\mathcal{F}$ with a probability $\phi_{\mathcal{F}}(\mathbf{g})$, while not transmitting any data on slot $mT, m = 0, 1, 2, ...$ We will show that the decentralized policy stabilized the system when it is can be stabilized by the aforementioned hypothetical policy. By Lemma 4.4.1, it suffices to prove that the decentralized policy stabilizes the system defined by $\tilde{\mathbf{q}}(m) = \mathbf{q}(mT)$ if the hypothetical policy renders it stable. Assume a mean arrival rate vector λ such that the system is stable under the hypothetical policy. Every timeslot $t \in \{mT+1, ..., mT+T-1\}$, the outcome of the decentralized policy is the F users with the greatest values of $\hat{q}_k(m)\mathbb{E}\left\{r_k(t)|g_k(t), F\right\}$. The drift of the quadratic Lyapunov function for the decentralized policy for the system $\tilde{\mathbf{q}}(m)$ is then (the inner expectations are with respect to the channel directions and the outer with respect to the channel magnitudes):

$$\begin{aligned}
\Delta V_{(d)}(\mathbf{q}) &\leq \tilde{B} + T\sum_{k=1}^K \tilde{q}_k(m)\lambda_k - (T-1)\mathbb{E}\left\{\sum_{k=1}^K \tilde{q}_k(m)\mathbb{E}\left\{\tilde{\mu}_k^{(d)}(m)|g_k, F\right\}|\hat{\mathbf{q}}(m)\right\} \\
&\leq (\tilde{B} + TKQ\lambda_k + (T-1)KR_LQ) + T\sum_{k=1}^K \hat{q}_k(m)\lambda_k \\
&- (T-1)\mathbb{E}\left\{\sum_{k=1}^K \hat{q}_k(m)\mathbb{E}\left\{\tilde{\mu}_k^{(d)}(m)|g_k, F\right\}|\hat{\mathbf{q}}(m)\right\} \\
&\leq \tilde{C} + T\sum_{k=1}^K \hat{q}_k(m)\lambda_k \\
&- (T-1)\mathbb{E}\left\{\sum_{k=1}^K \hat{q}_k(m) \sum_{\mathcal{F}:|\mathcal{F}=F|} \phi_{\mathcal{F}}(g_1, ..g_K)I_{\{k\in\mathcal{F}\}}\mathbb{E}\left\{\tilde{\mu}_k^{(d)}(m)|g_k, F\right\}\hat{\mathbf{q}}(m)\right\} \\
&\leq \tilde{C} + \sum_{k=1}^K (\hat{q}_k(m)(T\lambda_k - (T-1)\mu_k^*(F)) \leq \tilde{C} - \epsilon\sum_{k=1}^K \hat{q}_k(m),
\end{aligned}$$

for some $\epsilon > 0$, with $\tilde{C} = \tilde{B} + TKQ\lambda_k + (T-1)KR_LQ$ and the last inequality stems from the fact that λ is inside the stability region of the hypothetical policy. The drift gets negative for $||\hat{\mathbf{q}}||_1 > \tilde{C}/\epsilon$. Since $||\tilde{\mathbf{q}}||_1 > KQ + ||\hat{\mathbf{q}}||_1$, the drift is negative for $||\tilde{\mathbf{q}}||_1 \geq KQ + \frac{\tilde{C}}{\epsilon}$. From the Lyapunov-Foster criterion this implies that the system $\hat{\mathbf{q}}(m)$, therefore $\mathbf{q}(t)$, is stable under the decentralized policy if it is stable under a hypothetical policy that can achieve the stability region in the statement of this Lemma; therefore, this stability region is achievable by the decentralized policy.

We now proceed to show the converse, that is, if the system is stable under the decentralized policy for a mean arrival rate vector λ then this vector belongs in the set $\Lambda_d(F)$. Indeed, the set of users that are served at each timeslot $t = mT+1, ..., mT+T-1$ is a function on the realizations of the channel magnitude at slot t and the queue lengths $\mathbf{q}(mT)$. Denote this as $\mathcal{F}(\mathbf{q}, \mathbf{g})$. Since the channels are i.i.d. in time, the system $\tilde{\mathbf{q}}(m)$ is Markovian on a countable state space with a single communicating class. Stability then implies positive recurrence of the chain $\tilde{\mathbf{q}}(m)$, which further implies that there exist a (unique) stationary distribution, $\pi(\mathbf{q})$. The mean service rate (in bits per slot) user k gets is:

$$
\begin{aligned}
&\lim_{M\to\infty} \frac{1}{M} \sum_{m=0}^{M-1} \sum_{t=mT+1}^{(m+1)T-1} \mu_k(t) = (T-1) \sum_{\mathbf{q}\in\mathbb{Z}_+^K} \pi(\mathbf{q}) \mathbb{E}\left\{\mu_k|\mathbf{q}\right\} \\
&= (T-1) \sum_{\mathbf{q}\in\mathbb{Z}_+^K} \pi(\mathbf{q}) \int_0^\infty p_1(g_1)dg_1 ... \int_0^\infty p_K(g_K)dg_K \mathbb{E}\left\{\mu_k|\mathcal{F}(\mathbf{q},\mathbf{g}),\mathbf{g}\right\} \\
&= (T-1) \int_0^\infty p_1(g_1)dg_1 ... \int_0^\infty p_K(g_K)dg_K \sum_{\mathbf{q}\in\mathbb{Z}_+^K} \pi(\mathbf{q}) \mathbb{E}\left\{\mu_k|\mathcal{F}(\mathbf{q},\mathbf{g}),\mathbf{g}\right\} \\
&= (T-1) \int_0^\infty p_1(g_1)dg_1 ... \int_0^\infty p_K(g_K)dg_K \sum_{\mathcal{F}:|\mathcal{F}|=F} \sum_{\mathbf{q}\in\mathbb{Z}_+^K} \pi(\mathbf{q}) I_{\{\mathcal{F}=\mathcal{F}(\mathbf{q},\mathbf{g})\}} \mathbb{E}\left\{\mu_k|\mathcal{F},\mathbf{g}\right\}.
\end{aligned}
$$

Denote $\phi_{\mathcal{F}}(\mathbf{g})' = \sum_{\mathbf{q}\in\mathbb{Z}_+^K} \pi(\mathbf{q}) I_{\{\mathcal{F}=\mathcal{F}(\mathbf{q},\mathbf{g})\}}$. For every $\mathbf{g}$ this is a probability distribution over the sets $\mathcal{F}$. In addition, since the system is stable, the mean arrival rate should be less that the mean service rate for each user [15], therefore

$$
T\lambda_k < (T-1) \int_0^\infty p_1(g_1)dg_1 ... \int_0^\infty p_K(g_K)dg_K \phi_{\mathcal{F}}(\mathbf{g})' \mathbb{E}\left\{\mu_k|\mathbf{g}\right\},
$$

which implies that the vector of mean arrival rates λ indeed belongs to the set $\Lambda_d(F)$. □

Based on the above Lemma and the fact that according to the decentralized policy the BS selects every signalling slot $mT$the number of users to be scheduled in the next $T-1$ slots based on the state of the queue lengths at the beginning of slot mT and the channel statistics, the main result of this subsection follows:

Theorem 4.6.2. *The stability region of the decentralized policy is the convex combination of the regions given by Lemma 4.6.1 for every F, i.e.*

$$\Lambda_d = \mathcal{CH}\left\{\Lambda_d(1), ..., \Lambda_d(F), ..., \Lambda_d(F_{max})\right\}.$$

Proof. To begin with, note that this region is achievable by a randomized policy that selects a number of users F to get scheduled according to a properly defined probability distribution $\pi(F(m) = F)$ [7] and the exact set of users to be scheduled is determined by the contention procedure of the decentralized scheme. Assume any vector $\lambda \in \Lambda_d$. In this case, the re exist a probability distribution $\pi(F(m) = F)$ and $\epsilon > 0$ such that, for the mean service rate each user gets in each slot it holds

$$(T-1)\bar{\mu}_k - T\lambda_k \geq \epsilon. \tag{4.41}$$

We will show that the system under the decentralized policy is stable for this vector of mean arrival rates, meaning that Λ_d is achievable under the decentralized policy. Noticing that according to the decentralized policy the number $F(m)$ is selected in order to maximize the sum $\sum_{k=1}^{K} \hat{q}(m)\mathbb{E}\left\{\mu_k | F(m)\right\}$, we get for the Lyapunov drift of the system $\tilde{\mathbf{q}}(m)$:

$$\begin{aligned}
\Delta V_{(d)}(\mathbf{q}) &\leq \tilde{B} - T\sum_{k=1}^{K} \tilde{q}_k(m)\lambda_k + (T-1)\sum_{k=1}^{K} \tilde{q}(m)\mathbb{E}\left\{\mu_k^{(d)} | F(m)\right\} \\
&\leq \tilde{C} - \sum_{k=1}^{K} \hat{q}_k(m)(T\lambda_k - (T-1)\mathbb{E}\left\{\mu_k^{(d)} | F(m)\right\}) \leq \tilde{C} - \epsilon \left|\left|\hat{\mathbf{q}}(m)\right|\right|_1.
\end{aligned}$$

Following the same reasoning as in the proof of Lemma 4.6.1, this implies that the system is indeed stable under the decentralized policy for $\lambda \in \Lambda_d$.

To prove the converse, i.e. that every mean arrival rate vector λ for which the decentralized policy renders the system stable belongs to the set λ_d, we proceed in the same way as in the proof of Lemma 4.6.1. Since the system is stable for this vector, there exists a unique stationary distribution $\pi(\mathbf{q})$ of the markov chain that describes $\tilde{\mathbf{q}}(m)$. We have that $F(m)$ is in fact function of the queue lengths $\tilde{\mathbf{q}}(m)$ only, and we denote it by $F(\mathbf{q})$. Also, the mean service every user gets in a timeslot should be greater than the mean arrival rate. Based on that, and denoting $\bar{\mu}_k^{(d)}(F)$ the mean service (in bits per slot) user k gets

[7] Knowledge of the statistics of the arrival processes is needed along with the statistics of the channels to select this probability distribution

under the decentralized policy when $F(m) = F$ we have

$$T\lambda_k < (T-1) \sum_{\mathbf{q} \in \mathbb{Z}_+^K} \pi(\mathbf{q}) \bar{\mu}_k^{(d)}(F(\mathbf{q})),$$

which is indeed the convex hull of the sets $\Lambda_d(F)$. $\square$

The stability region of the system under the decentralized policy is thus the same as the biggest one achieved in the hypothetical case where all channel magnitude realizations were available to the scheduler, keeping the same timing overheads as in the decentralized policy. This shows why in the decentralized policy the users scheduled get high rates in *bits per channel use*. This is an advantage compared to the centralized policy, since in the latter a user with a bad channel realization may be scheduled. On the other hand, the decentralized policy comes with the disadvantage of spending one every T slots for signalling rather than data transmission and needing more time for exchange of control information in the data slot in order to implement the contention phase (though if the contention period can be made small and the decentralized policy tends to schedule fewer users than the centralized the additional overhead for the contention period may not pose a problem - however this is not guaranteed to happen in practice).

4.6.3 Mixed Policy

The mixed policy uses the same signalling structure as the centralized policy, however essentially switches between using the centralized and decentralized schemes every slot mT for the $T-1$ slots that follow. Note that if the mixed scheme operates in centralized mode selecting a set $\mathcal{F}(m)$, $T-1$ slots are used for data transmission, however only $(\beta_p + \beta F(m))$ channel uses are devoted to overhead for pilots and control signalling (since the IDs of the users to participate are fixed from the information at slot mT). Denote

$$\Lambda_c' = \left(1 - \frac{1}{T}\right) \mathcal{CH} \left\{ \frac{T_s - (\beta_p + \beta F)}{T_s - (1 + \beta_p + \beta F + \beta_c F)} \Lambda_c(\mathcal{F}) \right\}, \tag{4.42}$$

where

$$\Lambda_c(\mathcal{F}) = \left\{ \lambda \in \mathbb{R}_K^+ : \lambda_k < \mathbb{E}\left\{ (T_s - (\beta_p + \beta F + \beta_c F)) r_k(t) | F \right\} I_{\{k \in \mathcal{F}\}} \right\}, \tag{4.43}$$

i.e. the stability region if the set of users $\mathcal{F}$ was scheduled all the time according to the mixed policy. Then we have the following:

Theorem 4.6.3. *The stability region of the mixed policy is*

$$\Lambda_m = \mathcal{CH}\left\{ \Lambda_c', \Lambda_d \right\}.$$

The proof can be done in the same way as the ones in the previous subsections and is thus omitted to avoid repetition.

4.6.4 Comparison and Discussion

By selecting a high enough value of T, we can guarantee that the mixed scheme increases the stability region of the system. Denote

$$\hat{m} = \min_{1 \leq F \leq F_{max}} \left\{ \frac{T_s - (\beta_p + \beta F)}{T_s - (\beta_p + \beta F + \beta_c F + 1)} \right\}. \tag{4.44}$$

Then the following result holds:

Proposition 4.6.1. *A sufficient condition for the mixed scheme to have greater stability region than the centralized scheme is*

$$T > \frac{1}{1 - \hat{m}^{-1}}. \tag{4.45}$$

In this case, $\Lambda_m \supseteq \rho(T)\Lambda_c$ *with* $\rho(T) = \left(1 - \frac{1}{T}\right)\hat{m}$.

Proof. An immediate corollary of Theorem 4.6.3 is that $\Lambda_m \supseteq \Lambda_c'$. Note that, by the definition of this region, the signalling overhead required in a data slot is smaller than the one of the centralized policy. A sufficient condition, thus for $\Lambda_c' \supset \Lambda_c$ is

$$\left(1 - \frac{1}{T}\right) \min_{1 \leq F \leq F_{max}} \left\{ \frac{T_s - (\beta_p + \beta F)}{T_s - (\beta_p + \beta F + \beta_c F + 1)} \right\} > 1. \tag{4.46}$$

Replacing from (4.44), we get the stated result. □

For proving the expansion of the stability region by using the mixed policy, only the centralized mode of the mixed policy was used: The proof uses the fact that if the set of users to get scheduled is broadcasted periodically then, in each of the data slots, the signalling overhead is reduced (no control section is needed). Having the mixed policy select the same users as a centralized policy for most of the time can happen, for example, when the traffic patterns are such that a few users request very high rate or (in the extreme case) only one user requests nonzero rate. In these cases, it may be better to directly serve the users with the heavy traffic demands most of the time, in order to avoid the extra overhead for implementing the contention of the decentralized policy. In fact, in these cases the centralized policy might even perform better than the decentralized due to reduced overhead. On the other hand, the decentralized policy can get used in cases with more uniformly distributed traffic load. As seen in the system with the two users, using the decentralized policy along with the centralized one for the mixed policy can satisfy rate demands that the centralized policy alone could not. The result of Proposition 4.6.1 is thus a sufficient condition and expansion of the region in some directions of mean arrival rate vectors can be much higher than $\rho(T)$.

A last thing to note is that increasing the time T between two signalling slots enlarges the stability region, however this expansion is eventually bounded by system parameters. In addition, selecting a very big value of T can have a negative impact in terms of delays experienced by the users and can lead to slow convergence of the queueing system to its stationary distribution.

4.7 The case of OFDMA systems

In this Section we extend the analysis to systems using multiple orthogonal channels in frequency, i.e. OFDMA , on top of multiple antennas. We consider a system with N_c channels in frequency. Transmit power P is used for each channel, that is the total transmit power of the BS here is N_cP. The other parameters are the same as in the general description of Section 4.2 (i.e. N antennas, L rate levels). Channels are assumed to be independent in frequency and time, the channel state of user k on channel ν given as $\mathbf{h}_{k\nu}(t) = \sqrt{g_k}\hat{\mathbf{h}}_{k\nu}(t)$, where $\hat{\mathbf{h}}_{k\nu}(t) \sim \mathcal{CN}(0, \mathbf{I}_N)$. On each channel, Zero-Forcing precoding is employed to serve (potenitially) multiple users per time-frequency slot. Let $\mathcal{F}_\nu(t)$ the set of users that are scheduled at channel ν at timeslot t.

4.7.1 Single-antenna transmitter

We consider the case when the BS uses N_c frequency channels and one antenna first. Then, only one user is scheduled on each channel and each channel is characterized by its gain, $g_{k\nu}(t)$ for user k on channel ν at slot t only. For this subsection we relax the assumptions on the system model, considering channel gains that have arbitrary distributions, which is unknown to the BS and users. The BS here broadcasts, at slot mT, the quantized queue lengths $\hat{\mathbf{q}}(mT)$, as described in Section 4.3.2. At timeslot $t \in \{mT+1, ..., (m+1)T-1\}$, the BS broadcasts pilot signals of duration β_p channel uses to allow the users estimate their channels. Depending on the realization of the channel gains, the maximum rate user k can support on channel ν is $r_{k\nu}(g_{k\nu}(t))$. A contention period with duration of τ_c channel uses (and τ_c' μs) follows, where user k waits on every channel ν till time $\tau_{k\nu}'(t) = \frac{\tau_c'}{\hat{q}_k(m) r_{k\nu}(g_{k\nu}(t))}$ and transmits a signal for the rest of the contention period if no user has sent anything before in this channel. This is followed by a phase where every user that has sent a signal in every channel to send its ID and its achievable rate at each channel to the BS is used, consisting in $\beta_c = \frac{\log_2(K)}{R_0}$ and $\beta' = \frac{\log_2(L)}{R_0}$ channel uses, respectively. Then the BS serves the user that has gained the contention at each channel with rate $r_{k\nu}(g_{k\nu}(t))$. An illustration of the operation of the scheme is provided in Fig. 4.8.

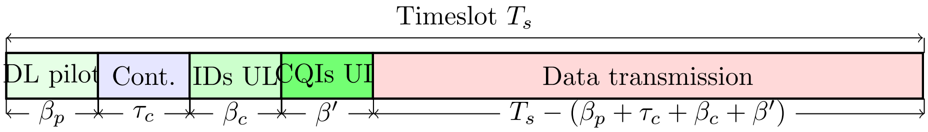

Figure 4.8: Operation of the decentralized scheme in a timeslot and channel when one antenna is used at the Base Station

The intuition behind the policy in the single-antenna transmitter case is that for every slot only one user will eventually get scheduled at each channel. Denoting Λ^* the stability region of the system under a policy where all channel realizations are assumed known to the BS at no cost in the beginning of every slot, we have the following result.

Theorem 4.7.1. *The stability region of the multi-user broadcast system with N_c channels and single antenna at the transmitter under the decentralized policy is*

$$\Lambda = \left(1 - \frac{1}{T}\right) \frac{T_s - (\beta_p + \tau_c + \beta')}{T_s} \Lambda^*.$$

Proof. In the ideal case where the BS knows all channel state realizations at no cost, the region Λ^* is achievable using the MaxWeight policy, that is, for each channel ν, schedule the user whose quantity $q_k(t) r_{k\nu}(t)$ is maximized [27]. Since we consider i.i.d. block fading channel with finite rate levels, we can denote $p_{kl} = \mathbb{P}\left\{S_l \leq \frac{g_{k\nu}(t)P}{\sigma^2} < S_{l+1}\right\}$, that is the probability that rate R_l bits per channel use is achievable for user k at timeslot t in channel ν (the channels of the same user are identically distributed). For the rest part of the proof, we will denote as "channel state" the realizations of the achievable rates in all channels of all users (instead of the realizations of the channel gains themselves). Denote $\mathcal{L}$ the set of all possible channel states. This set is finite since the possible rates at each channel are finite, and we will index its elements by l'. Under this notation, a necessary[8] and sufficient condition for $\lambda \in \Lambda^*$ is that there exist

[8] since we consider only the interiors of the regions

$0 \leq \phi_{k\nu}^{l'} \leq 1$ such that

$$\begin{aligned} \lambda_k &< \sum_{l' \in \mathcal{L}} p_{l'} \sum_{\nu=1}^{N_c} \phi_{k\nu}^{l'} r_{k\nu}^{l'} T_s, \forall k \in \mathcal{K} := \bar{\mu}_k^{SSS}, \\ 0 &< \sum_{k=1}^{K} \phi_{k\nu}^{l'} \leq 1, \forall l', \nu. \end{aligned} \tag{4.47}$$

In the above, $p_{l'}$ is the probability the channels are in state l', $r_{k\nu}^{l'}$ is the (maximum) rate user k can get on channel ν if the channels are in state l'. This is an application of the so-called Static Service Split Rule in [27] to our setting and implies that a hypothetical policy that properly scheduled user k on channel ν when the channel states are l' with a probability $\phi_{k\nu}^{l'}$ achieves the maximum possible stability region.

We now show that the region in the statement of the theorem is indeed achievable by the decentralized policy. We will prove stability for the system $\tilde{\mathbf{q}}(m) = \mathbf{q}(mT)$. From eq. (4.47), since one slot every T the BS does not transmit data, the channel and traffic processes are i.i.d. in time and not all channel uses are used for data transmission in a slot, it follows that for any vector $\lambda \in \Lambda$ there exist $0 \leq \phi_{k\nu}^{l'} \leq 1$ such that

$$T\lambda_k < (T-1)\frac{T_s - (\beta_p + \tau_c + \beta')}{T_s}\bar{\mu}_k^{SSS}, \forall k \in \mathcal{K}. \tag{4.48}$$

In addition, the Lyapunov drift for the system $\tilde{\mathbf{q}}(m)$ can be bounded as (refer to earlier Sections for details)

$$\Delta V(\mathbf{q}(m)) \leq \tilde{C} + T\sum_{k=1}^{K} \hat{q}_k(m)\lambda_k - (T-1)\sum_{k=1}^{K} \hat{q}_k(m)\mathbb{E}\left\{\mu_k|\hat{\mathbf{q}}(m)\right\}.$$

Notice that under the proposed contention scheme the user with the maximum value of $\hat{q}_k(t)r_{k\nu}(t)$ gets scheduled in each channel ν for every realization of the channel states, therefore we have

$$\begin{aligned} \Delta V(\mathbf{q}(m)) &\leq \tilde{C} - \sum_{k=1}^{K} \hat{q}_k(m)\left((T-1)\frac{T_s - (\beta_p + \tau_c + \beta')}{T_s}\bar{\mu}_k^{SSS} - T\lambda_k\right) \\ &\leq \tilde{C} - \epsilon\sum_{k=1}^{K} \hat{q}_k(m) \end{aligned}$$

for some $\epsilon > 0$, where the last inequality follows from (4.48). This implies that the system is stable, thus the decentralized policy achieves any arrival rate vector in the interior of Λ.

To finish we need to show that Λ is also an outer bound on the stability region of the decentralized policy. Indeed, condition (4.47) is also necessary, and we have that traffic arrives for all slots while the queues get service $T-1$ every T slots. This implies that the condition (4.48) is necessary for stability, therefore Λ is indeed an outer bound of the region. □

The theorem implies that in the case of one transmit antenna, the decentralized policy can achieve very close to the maximum possible stability region of the system. Note also that the result follows for any distribution of the channel states (and also for correlated channels among users and frequencies) and there is no need for the users and/or BS to know the channel statistics. This comes from the fact that in this setting only one user can be scheduled per channel.

4.7.2 Multiple-antenna transmitter

We now turn to the more general case with N antennas at the BS , assuming that the channel states are i.i.d. among different frequency channels of the same user. In this case, the policies of Section 4.3 can be carried over, running in parallel for each channel ν. Since the channels are assumed i.i.d. among frequencies for the same user, the parameters set by each policy (e.g. the sets $\mathcal{F}$ in the centralized policy and centralized mode of the mixed policy and the number of users F to get scheduled) are the same for every carrier. For the stability region of any policy π in the multiple channel case given its stability region in each carrier ν (i.e. if only this carrier was used to serve the users in the system) we have the following general result:

Theorem 4.7.2. *Let Λ^{π}_{ν} the stability region of the system using only carrier ν and Λ^{π} the region of the system using all carriers. Then the following holds:*

$$\Lambda^{\pi} = \oplus_{\nu=1}^{N_c} \Lambda^{\pi}_{\nu}. \tag{4.49}$$

Moreover, if channels in different frequencies are i.i.d. for the same user, the above reduces to

$$\Lambda^{\pi} = N_c \Lambda^{\pi}_{\nu}, \tag{4.50}$$

where Λ_{ν} is the stability region of one carrier.

Proof. The second part from the theorem follows directly from the first, so we will prove the first part only. We begin by showing that Λ^{π} is indeed achievable by policy π modified for the multicarrier case. Assume $\lambda \in \Lambda^{\pi}$. Then, we can write

$$\lambda = \sum_{\nu=1}^{N_c} \lambda_{\nu}, \text{with } \lambda_{\nu} \in \Lambda^{\pi}_{\nu}, \forall \nu = 1, ..., N_c, \tag{4.51}$$

for some proper λ_{ν}. In other words, there exist $0 \leq \phi_{\nu k} \leq 1$ such that $\sum_{\nu=1}^{N_c} \phi_{\nu k} = 1, \forall k \in \mathcal{K}$ such that $\forall \nu = 1, .., N_c$, $[a_{\nu 1}\lambda_1, ..., a_{\nu K}\lambda_K]^T \in \Lambda^{\pi}_{\nu}$.

This implies that a policy achieving this capacity region is a policy that creates a queue for every user at each channel ν, $q_{\nu k}(t)$ and routes the traffic arriving at time t for user k at queue ν with probability $\phi_{\nu k}$. These queues otherwise operate independently under the policy π applied to each channel ν. In this case, the system $q_{\nu k}(t)$ is stable and therefore every queue $q'_k(t) = \sum_{\nu=1}^{N_c} q_{\nu k}(t)$ is stable. Letting the average rate of user k at channel ν in this case be $\bar{\mu}_{\nu k}$, we have then

$$\bar{\mu}_{\nu k} > \lambda_{\nu k} = \phi_{\nu k}\lambda_k, \forall \nu, k. \tag{4.52}$$

On the other hand, every policy π from the ones presented in Section 4.3 tries to minimize some kind of Lyapunov drift based on the available information. For the mixed and decentralized policies, for the system $\tilde{\mathbf{q}}(m) = \mathbf{q}(mT)$, the above relationship becomes

$$(T-1)\bar{\mu}_{\nu k} > T\lambda_{\nu k} = T\phi_{\nu k}\lambda_k, \forall \nu, k, \tag{4.53}$$

therefore

$$\begin{aligned}
\Delta V_\pi(\mathbf{q}(m)) &\leq \tilde{C} + T\sum_{k=1}^{K}\lambda_k \hat{q}_k(m) - (T-1)\sum_{k=1}^{K}\hat{q}_k(m)\sum_{\nu=1}^{N_c}\mathbb{E}\left\{\tilde{\mu}^\pi_{\nu k}(m)|\hat{\mathbf{q}}(m)\right\} \\
&\leq \tilde{C} + T\sum_{k=1}^{K}\lambda_k \hat{q}_k(m) - (T-1)\sum_{k=1}^{K}\hat{q}_k(m)\sum_{\nu=1}^{N_c}\bar{\mu}_{\nu k} \\
&\leq \tilde{C} - \sum_{k=1}^{K}\hat{q}_k(m)\left(\epsilon + T\sum_{\nu=1}^{N_c}\phi_{\nu k}\lambda_k - T\lambda_k\right) = \tilde{C} - \epsilon\sum_{k=1}^{K}\hat{q}_k(m),
\end{aligned}$$

for some $\epsilon > 0$, due to (4.53). This implies that the system is stable under the policy π, thus π can achieve Λ^π.

For the centralized policy the achievability proof is essentially the same, just taking the system every slot t instead every slot mT and taking the actual queue lengths in the drift expressions.

To finish we need to prove the converse, that is that if the system is stable under π for a mean arrival rate vector λ then $\lambda \in \Lambda^\pi$. Then the system is an ergodic Markov chain, and since the service rates actually allocated to each user in each timeslot at each subcarrier depend on the queue lengths and the channel states, the limit $\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mu^\pi_{\nu k}(t)$ exists, for every user and carrier and it is equal to $\bar{\mu}^\pi_{\nu k}$ such that (since the system is assumed stable)

$$\lambda_k < \sum_{\nu=1}^{N_c}\bar{\mu}^\pi_{\nu k}, \forall k \in \mathcal{K}.$$

This means that, for some $\epsilon_k > 0$ we have $\forall k$: $\lambda_k = \sum_{\nu=1}^{N_c}(\bar{\mu}^\pi_{\nu k} - \frac{\epsilon_k}{N_c})$. However, at each carrier ν there should be $\bar{\mu}^\pi_\nu \in \Lambda^\pi_\nu$ (if not, these rates are not achievable in this carrier), therefore $\bar{\mu}^{\pi}_{\nu} - \frac{1}{N_c}\epsilon \in \Lambda^{\pi}_{\nu}$. Setting $\lambda_{\nu k} = \bar{\mu}^{\pi}_{\nu k} - \frac{\epsilon_k}{N_c}$, the mean arrival rate vector can be written as an element of Λ^{π}, therefore the converse is proved. $\square$

An application of the theorem above gives that, in the case of N_c carriers used, the stability regions of the centralized, decentralized and mixed policies are $N_c\Lambda_c$, $N_c\Lambda_d$, $N_c\Lambda_m$, respectively.

4.8 Trading signalling time for signalling bandwidth

In the above analysis we focused on the case where the quantized queue lengths and the number of users to be scheduled once every T timeslots, occupying the whole slot with this information. An alternative approach is to use an additional (low rate) control channel on which to broadcast this information.

More in detail, we add a downlink control channel of rate, say T_sR_c bits per time slot. Denote b the quantization bits per user; then the total number of signalling bits is $Kb + \log_2(K)$, transmission of which can be done in a time of $T = \frac{Kb+\log_2(K)}{T_sR_c}$ timeslots. The scheme here is to start broadcasting the signalling information of time mT at this slot and the users should use the most recent broadcasted information, i.e. at time $mT \leq t \leq (m+1)T - 1$ the users will use the broadcasted queue lengths and F of time $(m-1)T$. The error in the queue length estimation remains bounded and in addition all slots are now used for data transmission. It follows then that the stability region of this schemes will be the same as the corresponding schemes that take one slot for broadcast without the factor of $\left(1 - \frac{1}{T}\right)$. This statement can be proven using the same steps as the preceding analysis. For stability purposes, the rate of the added control channel can be arbitrarily low, with lower rate however probably deteriorating the delay performance of the scheme.

4.9 A dynamic threshold scheme for discrete time contention

In this Section we present a scheme to still leverage the fact that the receivers know their channel states (channel gain in the particular) for the case where contention is done in discrete time. We will assume throughout the Section that the channels of the users are i.i.d. with $\mathbf{h}_k(t) \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N), \forall k \in \{1, .., K\}$. In practice (where the users are in different distances from the BS), this can be achieved by appropriate power control.

More in detail, in slot mT the BS broadcasts an ordering of the users, a threshold g^* for the channel gain and a number of users to be scheduled, $F(m)$. In each of the next $T-1$ timeslots, after the downlink pilot, users access the channel in a TDMA manner on minislots of duration β_c channel uses each according to the ordering broadcasted at slot mT. When it is his turn to access the channel, each user sends a specified signal in the case its channel magnitude is above the threshold and does nothing otherwise. This phase ends when $F(m)$ users have signalled that they want to get included in the schedule or when all K users have accessed the channel. In the latter case, only the users that have signalled get scheduled and power is split only among them. Note that the duration of this control phase is generally variable, and the minislot for each user can have a duration of even one channel use. When this phase is over, the users that will be included in the schedule perform uplink training and the BS serves then using ZF precoding as described in Section 4.2.

The parameters to be optimized here are (i) the ordering of the users, (ii) the number of users, $F(m)$, to get scheduled and (iii) the threshold for the channel magnitudes, g^*, which should be in general a function of the queue lengths and F.

Under the setting of i.i.d. channels for the users, they will be ordered according to their queue lengths at (the beginning of) timeslot mT, with the user with the largest queue first.

We now turn to the impact of the threshold g^*. The probability that the channel magnitude of a user is above this threshold is given as

$$p(g^*) = \mathbb{P}\{g_k(t) > g^*\} = 1 - \frac{\gamma(g^*; N)}{\Gamma(N)} = \int_{g^*}^{+\infty} \frac{x^{N-1}e^{-x}}{\Gamma(N)} dx. \tag{4.54}$$

The above follows from the fact that since $\mathbf{h}_k(t) \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$, the magnitude follows a chi-squared distribution with $2N$ degrees of freedom. In addition, let $\mathbb{E}\left\{ r_k(t) \middle| F, g_k(t) > g^* \right\}$ be the mean rate in bits per channel use that user k gets if his channel magnitude is above the threshold and F users are scheduled in total. These can be calculated using the results of Section 4.4.1.

Intuitively, smaller threshold will cause the signalling phase to end faster, however users will worse channel condition will be selected. Assume a user ordering $\{k(1), k(2) ..., k(K)\}$ such that $q_{k(1)}(mT) \geq q_{k(2)}(mT) \geq ... \geq q_{k(K)}(mT)$. Then we have the following

Proposition 4.9.1. *The mean rate, $\bar{\mu}_{k(i)}(F, g^*)$ the i-th user in the ordering* *gets if the threshold is g^* and the BS signalled for F users to be scheduled is:*

$$\bar{\mu}_{k(i)}(1, g^*) = \\ [T_s - (\beta_p + \beta_c i + \beta)]^+ p(g^*)(1-p(g^*))^{i-1} \mathbb{E}\left\{ r_{k(i)}(t) \middle| 1, g_{k(i)}(t) > g^* \right\} \tag{4.55}$$

for $F(m) = 1$ and

$$\bar{\mu}_{k(i)}(F, g^*) = \mathbb{E}\left\{ r_{k(i)}(t) \middle| F, g_{k(i)}(t) > g^* \right\} p^F(g^*) \sum_{m=\max[F,i]}^{K} (1-p(g^*))^{m-F} \\ [T_s - (\beta_p + m\beta_c + F\beta)]^+ \sum_{f=\max\{0,F-1-m+i\}}^{\min\{F,i\}-1} \binom{i-1}{f} \binom{m-i}{F-(f+1)} \\ + \sum_{F'=1}^{F-1} \binom{K-1}{F'-1} [T_s - \beta_p - \beta_c K - \beta F']^+ p^{F'}(g^*)(1-p(g^*))^{K-F'} \\ \mathbb{E}\left\{ r_k(t) \middle| F', g_{k(i)}(t) > g^* \right\} \\ , \tag{4.56}$$

for $2 \leq F(m) \leq F_{max}$.

Proof. Please refer to Section 4.11.3 in the Appendix of this Chapter for the proof. ☐

The expected weight for $F(m) = F$ and threshold for the channel gain set to g^* then is given as

$$\mathbb{E}\left\{ \sum_{k=1}^{K} q_k(mT)\mu_k(t)z_k(t) \middle| F, g^* \right\} = \sum_{i=1}^{K} q_{k(i)}(mT)\bar{\mu}_{k(i)}(F, g^*).$$

From the above, the BS sets at slot mT the parameters $F(m), g^*(m)$ to be used in slots $mT+1, ..., mT+T-1$ as a solution to the following optimization problem:

$$\{F(m), g^*(m)\} = \arg \max_{g>0, 1 \leq F \leq F_{max}} \left\{ \sum_{i=1}^{K} q_{k(i)}(mT)\bar{\mu}_{k(i)}(F, g^*) \right\} \tag{4.57}$$

Denoting $\mathcal{G}$ the set of possible thresholds for the channel magnitude, the stability region achieved by this protocol is the following:

Theorem 4.9.1. *The stability region achieved under the dynamic threshold selection scheme is*

$$\Lambda_d^{th} = \left(1 - \frac{1}{T}\right) \mathcal{CH}_{\{F \in \{1,..,F_{max}\}, g^* \in \mathcal{G}\}} \{\bar{\mu}(F, g^*)\}.$$

The proof of the theorem is along the lines of the other proofs in this Chapter and omitted. Mixing this decentralized policy with a centralized one every T timeslots in the same way as the mixed policy described previously, we can achieve a stability region of $\mathcal{CH}\left\{\Lambda_c', \Lambda_d^{th}\right\}$.

4.10 Conclusions

In this Chapter we addressed the problem of user selection in a system were CSI is acquired by the BS via uplink training from the intended receivers. We have demonstrated that a feedback/training policy that combines decentralized schemes for user selection along with a centralized one can achieve greater stability region in the case of a MISO broadcast system. In addition, in the case of a SISO system the scheme can achieve an big fraction of the ideal stability region. These results suggest that, in future systems, decentralized methods should be considered for feedback and user scheduling along with the traditional centralized ones, at least for data-oriented services. We have also proposed a scheme based on a threshold for the channel magnitude in the case where contention is implemented in a slotted manner.

4.11 Appendix for Chapter 4

4.11.1 Proof of Proposition 4.4.1:

To begin with, we denote $k = k(i)$ for some $1 \leq i \leq F$ and define $\{k(j)\}_{j=1,..,F}$ the set of users that reported their CSI . For notational convenience we will drop the notation for dependence on t and F. For all other users except i, define the matrix of the vector of the respective channel corresponding to fast fading as $\hat{\mathbf{H}}_{k(i)} = \left[\hat{\mathbf{h}}_{k(1)}, ..., \hat{\mathbf{h}}_{k(i-1)}, \hat{\mathbf{h}}_{k(i+1)}, .., \hat{\mathbf{h}}_{k(F)}\right]$, the matrix of the respective directions, $\mathbf{U}_{k(i)} = \left[\mathbf{u}_{k(1)}, ..., \mathbf{u}_{k(i-1)}, \mathbf{u}_{k(i+1)}, .., \mathbf{u}_{k(F)}\right]$, the matrix of large scale channel gains $\bar{\mathbf{G}}_{k(i)} = \mathrm{diag}\left\{\bar{g}_{k(1)}, .., \bar{g}_{k(i-1)}, \bar{g}_{k(i+1)}, ..., \bar{g}_{k(F)}\right\}$ and finally the diagonal matrix containing the instantaneous channel gains, $\mathbf{G}_{k(i)} = \mathrm{diag}\left\{g_{k(1)}, .., g_{k(i-1)}, g_{k(i+1)}, ..., g_{k(F)}\right\}$. Using these notations we can write:

$$\mathbf{H}_{k(i)} = \hat{\mathbf{H}}_{k(i)}\bar{\mathbf{G}}_{k(i)} = \mathbf{U}_{k(i)}\mathbf{G}_{k(i)}. \tag{4.58}$$

Replacing the above in (4.5), we get that the SNR at user $k(i)$ then can be written as

$$SNR_{k(i)} = \frac{g_{k(i)}P}{\sigma^2 F}\mathbf{u}_{k(i)}^H\left(\mathbf{I}_N - \mathbf{U}_{k(i)}\left(\mathbf{U}_{k(i)}^H\mathbf{U}_{k(i)}\right)^{-1}\mathbf{U}_{k(i)}^H\right)\mathbf{u}_{k(i)}. \tag{4.59}$$

However, we have from (4.58) that

$$\mathbf{U}_{k(i)} = \hat{\mathbf{H}}_{k(i)}\bar{\mathbf{G}}_{k(i)}\mathbf{G}_{k(i)}^{-1} = \hat{\mathbf{H}}_{k(i)}\mathbf{D}_{k(i)}, \tag{4.60}$$

where we have defined $\mathbf{D}_{k(i)} = \bar{\mathbf{G}}_{k(i)}\mathbf{G}_{k(i)}^{-1} = \mathrm{diag}\left\{\frac{\bar{g}_{k(1)}}{g_{k(1)}}, .., \frac{\bar{g}_{k(i-1)}}{g_{k(i-1)}}, \frac{\bar{g}_{k(i+1)}}{g_{k(i+1)}}, ..., \frac{\bar{g}_{k(F)}}{g_{k(F)}}\right\}$. In addition, since $\hat{\mathbf{H}}_{k(i)}$ is a matrix with $F-1$ i.i.d. $\mathcal{CN}(0, \mathbf{I}_N)$ random vectors as columns, we can write its Singular Value Decomposition as [102]

$$\hat{\mathbf{H}}_{k(i)} = \mathbf{V\Sigma\Delta}, \tag{4.61}$$

where $\mathbf{V} \in \mathbb{C}^{N\times N}$ and $\mathbf{\Delta} \in \mathbb{C}^{(F-1)\times(F-1)}$ are isotropically distributed unitary matrices (i.e. Haar matrices) and $\mathbf{\Sigma} \in \mathbb{C}^{N\times(F-1)}$ is the matrix containing the singular values of $\hat{\mathbf{H}}_{k(i)}$. Replacing in the SNR expression (4.59) and noting that $\mathbf{VV}^H = \mathbf{I}_N$ we eventually get

$$\begin{aligned} SNR_{k(i)} &= \frac{g_{k(i)}P}{\sigma^2 F}\mathbf{u}_{k(i)}^H\left(\mathbf{VV}^H - \mathbf{V\Sigma}\left(\mathbf{\Sigma}^H\mathbf{\Sigma}\right)^{-1}\mathbf{\Sigma}^H\mathbf{V}^H\right)\mathbf{u}_{k(i)} \\ &= \frac{g_{k(i)}P}{\sigma^2 F}\mathbf{u}_{k(i)}^H\mathbf{V}\left(\mathbf{I}_N - \mathbf{\Sigma}\left(\mathbf{\Sigma}^H\mathbf{\Sigma}\right)^{-1}\mathbf{\Sigma}^H\right)\mathbf{V}^H\mathbf{u}_{k(i)}. \end{aligned} \tag{4.62}$$

Defining now

$$\begin{aligned} \mathbf{v} &= \mathbf{V}^H\mathbf{u}_{k(i)}, \\ \mathbf{A} &= \begin{bmatrix} \mathbf{0}_{F-1,F-1} & \mathbf{0}_{F-1,N-F+1} \\ \mathbf{0}_{N-F+1,F-1} & \mathbf{I}_{N-F+1} \end{bmatrix} \end{aligned}$$

we have eventually

$$SNR_{k(i)} = \frac{g_{k(i)}P}{\sigma^2 F}\mathbf{v}^H\mathbf{A}\mathbf{v} = \frac{g_{k(i)}P}{\sigma^2 F}\sum_{j=F+1}^{N}|u_j|^2. \tag{4.63}$$

Note that the summation is over the squared magnitudes of the $N-F+1$ last components of the vector $\mathbf{v}$. Since $\mathbf{V}$ is a Haar matrix and $\mathbf{u}_{k(i)}$ is unitary, $\mathbf{v} \in \mathbb{C}^{N\times 1}$ is a random unitary isotropic vector with distribution [103]

$$p(\mathbf{v}) = \frac{\Gamma(N)}{\pi^N}\delta\left(||\mathbf{v}||^2 - 1\right), \tag{4.64}$$

where $\delta(.)$ denotes the Dirac function. We now define the vectors $\psi_1 \in \mathbb{C}^{(F-1)\times 1}$ and $\psi_2 \in \mathbb{C}^{(N-F+1)\times 1}$ such that $\psi_2 = [v_{F+1}, ..., v_N]^T$ and $\mathbf{v} = [\psi_1^T, \psi_2^T]^T$. In this case, we can write the SNR as

$$SNR_{k(i)} = \frac{g_{k(i)}P}{\sigma^2 F}\psi_2^H\psi_2. \tag{4.65}$$

In addition, note that $||\mathbf{v}||^2 = ||\psi_1||^2 + ||\psi_2||^2$ and the channel coefficient corresponding to each antenna is independent of the coefficients corresponding to the other antennas.

We will deal with the probability distribution function of the SNR (4.63) first. From the aforementioned expression, if follows that

$$p_{SNR_{k(i)}}(s|g_{k(i)}, F) = \frac{F\sigma^2}{g_{k(i)}P} p_{\psi_2^H \psi_2}\left(\frac{F\sigma^2}{g_{k(i)}P}s\right) \tag{4.66}$$

To proceed further, we note that the p.d.f. of $\psi_2^H \psi_2$ is in fact the probability that a vector is drawn from the distribution of unitary isotropic vectors with the sums of the squared magnitudes of its $N - F + 1$ last elements equal to $\psi_2^H \psi_2$, thus:

$$\begin{aligned} p_{\psi_2^H \psi_2}(x) &= \int_{\mathbf{v}\in\mathbb{C}^N:||\mathbf{v}||=1,\sum_{k=F}^{N}|v_k|^2=x} p(\mathbf{v})d\mathbf{v} = \int_{\mathbf{v}\in\mathcal{C}^N:\sum_{k=F}^{N}|v_k|^2=x} \frac{\Gamma(N)}{\pi^N}\delta\left(\mathbf{v}^H\mathbf{v}-1\right)d\mathbf{v} \\ &= \frac{\Gamma(N)}{\pi^N}\int_{\psi_2\in\mathbb{C}^{N-F+1}} d\psi_2\delta\left(\psi_2^H\psi_2 - x\right)\int_{\psi_1\in\mathbb{C}^{F-1}} d\psi_1\delta\left(\psi_1^H\psi_1+\psi_2^H\psi_2-1\right) \\ &= \frac{\Gamma(N)}{\pi^N}\left(\int_{\psi_2\in\mathbb{C}^{N-F+1}}\delta\left(\psi_2^H\psi_2-x\right)d\psi_2\right)\left(\int_{\psi_1\in\mathbb{C}^{F-1}}\delta\left(\psi_1^H\psi_1-(1-x)\right)d\psi_1\right) \end{aligned} \tag{4.67}$$

In order to calculate the integrals we make use the Fourier transform of the Dirac function, a method used also in [104]. We have the following general result (it is similar to a result in [104] but we present its proof here for completeness) :

Lemma 4.11.1. *For $M \in \mathbb{Z}_+$ and $r > 0$ it holds*

$$\int_{\mathbf{u}\in\mathbb{C}^M}\delta\left(\mathbf{u}^H\mathbf{u}-r\right)d\mathbf{u} = \frac{\pi^M}{\Gamma(M)}r^{M-1}.$$

Proof. For this proof, i will denote the imaginary unit (i.e. $i^2 = -1$). We begin by replacing the Dirac function with its Fourier representation, that is $\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega e^{i\omega x}$ and we get, for (any) $\alpha > 0$,

$$\begin{aligned} \int_{\mathbf{u}\in\mathbb{C}^M}\delta\left(\mathbf{u}^H\mathbf{u}-r\right)d\mathbf{u} &= \int_{\mathbf{u}\in\mathbb{C}^M} d\mathbf{u}\frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega e^{i\omega\left(\mathbf{u}^H\mathbf{u}-r\right)} \\ &= \frac{1}{2\pi}\int_{\mathbf{u}\in\mathbb{C}^M} d\mathbf{u}e^{-\alpha\mathbf{u}^H\mathbf{u}}e^{+\alpha\mathbf{u}^H\mathbf{u}}\int_{-\infty}^{+\infty} d\omega e^{i\omega\left(\mathbf{u}^H\mathbf{u}-r\right)} \\ &= \frac{e^{\alpha r}}{2\pi}\int_{\mathbf{u}\in\mathbb{C}^M} d\mathbf{u}e^{-\alpha\mathbf{u}^H\mathbf{u}}\int_{-\infty}^{+\infty} d\omega e^{i\omega\left(\mathbf{u}^H\mathbf{u}-r\right)} \qquad (4.68) \\ &= \frac{e^{\alpha r}}{2\pi}\int_{-\infty}^{+\infty} d\omega e^{-i\omega r}\int_{\mathbf{u}\in\mathbb{C}^M} d\mathbf{u}e^{-\mathbf{u}^H\mathbf{u}(\alpha-i\omega)} \\ &= \frac{e^{\alpha r}}{2\pi}\int_{-\infty}^{+\infty} d\omega e^{-i\omega r}\frac{\pi^M}{(\alpha-i\omega)^M} \\ &= e^{\alpha r}\pi^M\left(\frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega e^{i\omega r}\frac{1}{(\alpha+i\omega)^M}\right). \qquad (4.69) \end{aligned}$$

Line (4.68) is obtained by noting that the integrand with respect to $\mathbf{u}$ is nonzero only for $\mathbf{u}^H\mathbf{u} = r$. The term in the parenthesis in (4.69) is the inverse Fourier transform of the function $\frac{1}{(\alpha+i\omega)^M}$. Noting that the Fourier transform of $e^{-\alpha r}I_{r\geq 0}$ is $\frac{1}{\alpha+i\omega}$ and using the properties of Fourier transforms (mainly the properties about their derivatives and linearity) we get, for $r > 0$

$$\int_{\mathbf{u}\in\mathbb{C}^M} \delta\left(\mathbf{u}^H\mathbf{u} - r\right) d\mathbf{u} = e^{\alpha r}\pi^M \frac{r^{M-1}e^{-\alpha r}}{(M-1)!} = \frac{\pi^{M-1}}{(M-1)!}r^{M-1},$$

which, since for any positive integer M there is $(M-1)! = \Gamma(M)$, completes the proof. $\square$

Applying Lemma 4.11.1 in equation (4.67) we get eventually

$$p_{\psi_2^H\psi_2}(x) = \frac{\Gamma(N)}{\Gamma(F-1)\Gamma(N-F+1)}(1-x)^{N-F}x^{F-2} = \frac{(1-x)^{N-F}x^{F-2}}{B(N-F+1,F-1)}. \tag{4.70}$$

Using (4.66) and replacing with (4.70) we have

$$\begin{aligned}
\mathbb{P}\left\{SNR_{k(i)} > S|g_{k(i)}, F\right\} &= \int_S^{\frac{Pg_{k(i)}}{\sigma^2 F}} p_{SNR_{k(i)}}(s|g_{k(i)}, F)ds \\
&= \int_S^{\frac{Pg_{k(i)}}{\sigma^2 F}} \frac{F\sigma^2}{g_{k(i)}P} p_{\psi_2^H\psi_2}\left(\frac{F\sigma^2}{g_{k(i)}P}s\right) ds \\
&= \int_{\frac{F\sigma^2}{Pg_{k(i)}}S}^1 p_{\psi_2^H\psi_2}(x)dx \\
&= \frac{1}{B(N-F+1,F-1)}\int_{\frac{F\sigma^2}{Pg_{k(i)}}S}^1 (1-x)^{N-F}x^{F-2}dx,
\end{aligned}$$

which is the stated result.

4.11.2 Proof of Lemma 4.4.2:

From the evolution equation for the sampled queue lengths we have

$$\begin{aligned}
\tilde{q}_k^2(m+1) = \Bigg(&\tilde{q}_k(m) + \sum_{t=0}^{T-1} a_k(mT+t) - \sum_{t=1}^{T-1} z_k(mT+t)\mu_k(mT+t) + \\
&\sum_{t=1}^{T-1} y_k(mT+t)\Bigg)^2 \\
&\leq \left(\tilde{q}_k(m) + \sum_{t=0}^{T-1} a_k(mT+t) - \sum_{t=1}^{T-1} z_k(mT+t)\mu_k(mT+t)\right)^2 \\
&= \tilde{q}_k^2(m) + \left(\sum_{t=0}^{T-1} a_k(mT+t)\right)^2 + \left(\sum_{t=1}^{T-1} z_k(mT+t)\mu_k(mT+t)\right)^2 \\
&+ 2\tilde{q}_k(m)\sum_{t=0}^{T-1} a_k(mT+t) - 2\tilde{q}_k(m)\sum_{t=1}^{T-1} z_k(mT+t)\mu_k(mT+t) \\
&\leq \tilde{q}_k^2(m) + T^2 A_{max}^2 + (T-1)^2 R_L^2 \\
&- 2\tilde{q}_k(m)\sum_{t=0}^{T-1} a_k(mT+t) + 2\tilde{q}_k(m)\sum_{t=1}^{T-1} z_k(mT+t)\mu_k(mT+t).
\end{aligned}$$

From the above, setting $\tilde{B} = K(T^2 A_{max}^2 + (T-1)^2 R_L^2)$ and taking expectations it follows that

$$\begin{aligned}
\Delta V(\tilde{\mathbf{q}}(m)) \leq \tilde{B} &+ \mathbb{E}\left\{\sum_{k=1}^{K} 2\tilde{q}_k(m)\sum_{t=0}^{T-1} a_k(mT+t)|\tilde{q}_k(m)\right\} \\
&- \mathbb{E}\left\{\sum_{k=1}^{K} 2\tilde{q}_k(m)\sum_{t=1}^{T-1} z_k(mT+t)\mu_k(mT+t)|\tilde{q}_k(m)\right\} \\
&= \tilde{B} + 2\sum_{k=1}^{K}\tilde{q}_k(m)\sum_{t=0}^{T-1}\mathbb{E}\{a_k(mT+t)\} \\
&- 2\sum_{k=1}^{K}\tilde{q}_k(m)\sum_{t=1}^{T-1}\mathbb{E}\{z_k(mT+t)\mu_k(mT+t)|\tilde{\mathbf{q}}(m)\} \\
&= \tilde{B} + 2T\sum_{k=1}^{K}\tilde{q}_k(m)\lambda_k \\
&- 2\sum_{k=1}^{K}\tilde{q}_k(m)\sum_{t=1}^{T-1}\mathbb{E}\{z_k(mT+t)\mu_k(mT+t)|\tilde{\mathbf{q}}(m)\}
\end{aligned}$$

Where the last equality follows from the fact that the arrival processes are i.i.d. in time and independent of anything else. Moreover, we have that the schedule $z_k(mT+t)$ and the rate of the user in the slot $\mu_k(mT+t)$ depend only on the queue length vector $\tilde{\mathbf{q}}(m)$ and the realizations of the channels and not on the time index. This implies that the product $z_k(mT+t)\mu_k(mT+t)$ has the same distribution for every slot between $mT+1 \leq \tau \leq m(T+1)$. Defining $\tilde{\mu}_k(m)$ as this final rate (including the scheduling decision) user k gets, for a given scheme it is as if we have $(T-1)$ independent copies of this random variable (with probability distribution over the probability distribution of the channels), hence the statement follows.

4.11.3 Proof of Proposition 4.9.1:

Proof of equation (4.55): Notice that here only one user is to be scheduled (since $F(m)=1$). This means that the i-th user in the ordering gets scheduled if (i) his channel magnitude is above the threshold *and* (ii) the channel magnitudes of the $i-1$ users before him are below the threshold. In addition, if this user is scheduled the contention period stops right after, i.e. lasts for i minislots.

Proof of equation (4.56): We now deal with the case where $2 \leq F(m) = F \leq F_{max}$. The mean rate of the i-th user in the ordering can be written as follows:

$$\begin{aligned}\bar{\mu}_{k(i)}(F,g^*) &= \mathbb{P}\left\{F \text{ users above threshold}, k(i) \in \mathcal{F}\right\} \mathbb{E}\left\{\bar{\mu}_{k(i)} | F \text{ above}, k(i) \in \mathcal{F}\right\} \\ &+ \sum_{F'=1}^{F-1} \mathbb{P}\left\{F' \text{ users above threshold}, k(i) \in \mathcal{F}\right\} \mathbb{E}\left\{\bar{\mu}_{k(i)} | F' \text{ above}, k(i) \in \mathcal{F}\right\} \\ &:= \hat{\mu}_i(F) + \sum_{F'=1}^{F-1} \hat{\mu}_i(F').\end{aligned} \tag{4.71}$$

For the rest of the proof, we will denote the event that user $k(i)$ is scheduled as $k(i) \in \mathcal{F}$. For the first term of the above equation, which corresponds to the event that $F(m) = F$ users get scheduled in the end, we have

$$\begin{aligned}\hat{\mu}_i(F) = \sum_{m=F}^{K} \mathbb{P}\left\{M = m, k(i) \in \mathcal{F}, |\mathcal{F}| = F\right\} \left[T_s - (\beta_p + \beta F + \beta_c m)\right]^+ \\ \mathbb{E}\left\{ r_{k(i)}(t) \middle| F, g_{k(i)}(t) > g^* \right\},\end{aligned} \tag{4.72}$$

where M denotes the duration of the contention period (i.e. how many minislots are used till F users are found with the channel magnitude above the threshold). Since $k(i)$ should be in the set of users that are scheduled, the contention period should not stop before his minislot, that is

$$\mathbb{P}\left\{M = m, k(i) \in \mathcal{F}, |\mathcal{F}| = F\right\} = 0, \forall 0 \leq m < i. \tag{4.73}$$

To proceed further, we note that the event in the probability in equation (4.72) is equivalent to the union of events where (i) the channel magnitude of user $k(i)$

is above the threshold and (ii) f users with lower order than i, that is between and including $k(1)$ and $k(i-1)$ and $F-(f+1)$ between and including $k(i+1)$ and $k(m)$ have channel magnitudes above the threshold, for all values of f. The values f is allowed to take should satisfy the following properties:

$$\begin{aligned} &0 \leq f \leq i-1 \\ &f \leq F-1 \\ &F-(1+f) \leq m-i \implies f \geq F-1-(m-i). \end{aligned} \tag{4.74}$$

The first condition in (4.74) comes from the fact that the number of users before the i-th user is $i-1$, the second from the fact that in order for $k(i)$ to be scheduled, less than F users higher in the ranking should have channel magnitudes above the threshold and the third because there are left $m-i$ users in the ranking after user $k(i)$. We thus have

$$\begin{aligned} &\mathbb{P}\{M=m, k(i) \in \mathcal{F}, |\mathcal{F}|=F\} = \\ &p(g^*) \sum_{f=\max\{0,F-1-m+i\}}^{\min\{F,i\}-1} \mathbb{P}\Big\{ f \text{ in } \{k(1),...,k(i-1) \text{ above threshold },\\ &F-(f+1) \text{ in } \{k(i+1),...,k(m) \text{ above threshold } \Big\}, \end{aligned}$$

which, since the channels are i.i.d. among users reduces to

$$\begin{aligned} &\mathbb{P}\{M=m, k(i) \in \mathcal{F}, |\mathcal{F}|=F\} = \\ &p(g^*) \sum_{f=\max\{0,F-1-m+i\}}^{\min\{F,i\}-1} \mathbb{P}\Big\{ f \text{ out of } i-1 \text{ above threshold } \Big\} \\ &\mathbb{P}\Big\{ F-(f+1) \text{ out of } m-i \text{ above threshold } \Big\}. \end{aligned}$$

The event in the first term inside the sum happens in $\binom{i-1}{f}$ possible ways with probability $p^f(g^*)(1-p(g^*))^{i-1-f}$ each, while the event in the second term happens in $\binom{m-i}{F-(f+1)}$ ways, with probability $p^F-(f+1)(g^*)(1-p(g^*))^{m-i-(F-(f+1))}$ each. Replacing and taking into account that $F \leq m$ (all F users should get scheduled) and eq. (4.73) we get eventually

$$\mathbb{P}\{M=m, k(i) \in \mathcal{F}, |\mathcal{F}|=F\} =$$

$$\begin{cases} \sum_{f=\max\{0,F-1-m+i\}}^{\min\{F,i\}-1} \binom{i-1}{f}\binom{m-i}{F-(f+1)} p(g^*)^F(1-p(g^*))^{m-F}, \max\{F,i\} \leq m \leq K \\ 0, 0 \leq m < \max\{F,i\} \end{cases} \tag{4.75}$$

We now turn to the second term, i.e. the sum $\sum_{F'=1}^{F-1} \hat{\mu}_i(F')$. This is in fact due to the probability that less than $F(m)$ users can have channel magnitude above the threshold. The event that exactly $F' < F$ users have channel magnitude above the threshold and user $k(i)$ is scheduled is the same as user $k(i)$ having channel magnitude over the threshold and exactly $F' - 1$ out of the remaining $K - 1$ users do. This event can happen in $\binom{K-1}{F'-1}$ combinations, each having a probability of $p(g^*)p^{F'-1}(g^*)(1 - p(g^*))^{K-F'}$. On the other hand, if less than $F(m)$ users are above the threshold then all K minislots are used, therefore the contention period lasts for $\beta_c K$ channel uses. Finally, F' users participate in the uplink training. We then have

$$\begin{aligned}\hat{\mu}_i(F') = &\binom{K-1}{F'-1} p^{F'}(g^*)(1 - p(g^*))^{K-F'} \\ &[T_s - (\beta_p + \beta_c K + \beta F')]^+ \mathbb{E}\left\{ r_{k(i)}(t) \middle| F', g_{k(i)}(t) > g^* \right\}.\end{aligned} \tag{4.76}$$

The result in eq. (4.56) follows combining (4.71) with (4.75), (4.72) and (4.76).

Chapter 5

Joint Feedback and Scheduling in TDD Multichannel Downlink Systems

5.1 Introduction

In this Chapter we address the problem of joint feedback and scheduling in multiuser downlink systems employing parallel channels to serve the users. This setting corresponds to single cell OFDMA systems and/or systems with multiple antennas at the base stations where orthonormal beamforming is used. Both schemes are actually implemented in the LTE standards [11] and can offer a substantial increase in the system's performance. In order to fully exploit the potential of these techniques, knowledge of the user's channel states is needed, thus resulting in a big overhead. However, at the end, only one user will be scheduled in each channel. Based on this observation, for a single channel system where the goal is to maximize spectral efficiency, the authors in [57] show that having all users feeding back their channel state is not really needed and propose a threshold-based policy that reduced feedback load while still achieving the benefits of multiuser diversity. This scheme is further enhanced in [105]. Other ideas to limit the feedback needed include grouping subcarriers and sending a single CQI for them [58] and/or reporting only the strongest channels of each user [58], [59]. In addition, some schemes based on random access using a collision channel for feedback e.g. [60], [61], [62] have been proposed, by tuning the transmission probabilities and thresholds appropriately. However, the main focus of these works is spectral efficiency/sum rate and they do not take into account the incoming traffic processes of the users.

Regarding the effect of feedback on stability performance, the authors in [56] study the problem of deciding which subset of users to collect feedback from, while the authors in [54] investigate the achievable stability region in a multichannel system with infrequent channel measurements. In these works channel statistics are assumed known. Moreover, in [52], a CSMA -based scheme is presented for channel state feedback and in [63] the authors devise a feedback scheme for a multiuser MIMO downlink employing orthonormal beamforming. In these cases however the authors do not take into account the fact that the base station must wait for some time in the slot before the feedback can be used. Assuming channel statistics are known, the authors in [64] propose a heuristic feedback scheme with two feedback slots based on the idea of maximum quantile scheduling. Furthermore, in [65] it is shown that for a system of L carriers with FDD mode for feedback, the base station needs to acquire at least $\Theta(L)$ channel realizations each time slot to obtain very close to the biggest achievable stability region. In [66], a TDD mode of feedback is used: the base station requests the users to feed back their channel states but each procedure is centralized and takes up a portion of the time slot. Based on optimal stopping theory and assuming that the distributions of the channel gains are known to the base station, the authors derive the general properties of the *centralized* optimal probing policy and completely characterize it in some special cases. Finally, for the same model, the authors in [67, 68] have recently proposed a simple feedback scheme for a single channel system. This scheme, termed Selective Scheduling and Feedback (SFF), sets as threshold the rate of the user with the maximum queue length and requires no knowledge of channel and traffic statistic. It is shown to guarantee greater stability region than a scheme where all channels are probed. In multi-carrier systems, the probing problem is more challenging since a user may be scheduled on a subset of channels and therefore each user needs to feed back the CQIs of a subset (as small as possible) of its channels. Applying directly the aforementioned schemes to multi-carrier systems may not result in a good stability region because the number of users feeding back on each channel might be still big. This poses more of a problem as the number of users in the cell increases.

In this Chapter, we focus on the downlink of a multichannel single cell system with feedback in TDD mode. In fact, in Section 4.7.1 we have shown that if we assume that a contention scheme working on continuous time with very short contention signals is used, a very big fraction of the stability region of the ideal case (channel realizations known to the base station at no cost) can be achieved. However, these assumptions can be rather strong in practical systems.In this chapter, we propose two schemes for both of which a threshold for the achievable rate of the channel is adjusted by the base station according to the queue lengths of the users, in a way similar to [68]. We examine two approaches: in the first one, a user whose achievable rate exceeds the threshold feeds back with a properly defined probability and in the second one each user whose achievable rate is above the threshold feeds back, but the base station can decide when to stop the feedback procedure. We illustrate both by simulations and analysis that these schemes outperform the SSF scheme of [68] in terms of achievable stability region.

The rest of the chapter is organized as follows: In Section 5.2 we present the system model. The randomized and stopping-based algorithms are presented in Sections 5.3 and 5.4, respectively. These Sections include the descriptions and analysis of these algorithms. Finally, Section 5.5 concludes the chapter.

5.2 Preliminaries

We consider a single cell system where a base station serves K users using N channels, assumed to be randomly time varying, i.i.d. across time. This can model the case of OFDMA downlink schemes with N carriers or the case where the base station is equipped with N antennas and orthonormal beamforming is used (in the latter case a "channel" is a beamforming vector). Time is slotted. Let $r_{kn}(t)$ be the achievable rate for user k at channel n at timeslot t (in bits per timeslot duration). This rate is assumed to belong to a set of finite values, $\{R_1, .., R_L\}, R_1 = 0, R_{l+1} > R_l$, which is the case in practical systems, as a finite number of modulation and coding schemes is used. Also the rates are independent from each other and across users, but not necessarily identically distributed. Each user $i \in \{1, ..., K\}$ is associated with a randomly incoming traffic process $a_i(t)$ with mean rate λ_i. Incoming traffic processes are i.i.d. across time, independent across users and independent with respect to the channel processes. For the MAC layer, the base station maintains a different queue for each user, whose queue length at time slot t is denoted $q_i(t)$.

Define now the weight of user k at channel n as $W_{kn}(t) = q_t(t)r_{kn}(t)$. As explained in Chapter 2, if all channel realizations are known, the MaxWeight scheduler, where at each channel n the user with the maximum weight is scheduled achieves the biggest stability region possible. However, we consider feedback in TDD mode and the slot divided in minislots of duration βT_s. At each minislot, at each channel the base station can either request a user to feed back on this channel, broadcast information or let users feed back in a decentralized way. In channel n, let $M_n(t)$ be the number of minislots used by the feedback procedure at time slot t; then, if user $k^*(n)$ is scheduled, it will receive $(1-\beta M_n(t))r_{k^*(n)n}(t)$ bits at timeslot t. Define $z_{kn}(t)$ the scheduling decision at time slot t (i.e. $z_{kn}(t)=1$ if user k is scheduled on channel n at time slot t and zero otherwise). As mentioned in the introduction of this Section, we will compare our scheme with the scheme of [68] applied in multiple carriers (will be referred to as "SSF" in the rest of the paper, standing for "Selective Scheduling and Feedback" [67, 68]). Variables with a tilde will be the quantities corresponding to the proposed schemes, while variables denoted with normal letters will correspond to the SSF scheme. Note then that $z_{kn}(t)$ is the same schedule as MaxWeight scheduling when all the channels were known [68].

Define the following quantities under the SSF and any of the two scheduling and feedback schemes, respectively:

$$f(\mathbf{q}(t)) = \mathbb{E}\left\{\sum_{n=1}^{N}[1-\beta M_n(t)]^+ \sum_{i=1}^{K} q_i(t) r_{in}(t) z_{in}(t) \middle| \mathbf{q}(t)\right\} \tag{5.1}$$

$$\tilde{f}(\mathbf{q}(t)) = \mathbb{E}\left\{\sum_{n=1}^{N}[1-\beta \tilde{M}_n(t)]^+ \sum_{i=1}^{K} q_i(t) r_{in}(t) \tilde{z}_{in}(t) \middle| \mathbf{q}(t)\right\}. \tag{5.2}$$

Note that these quantities correspond to the negative part of the drift of the quadratic Lyapunov function under the aforementioned schemes. Our proposed schemes are based on the following result [53] (see also [65, 68, 106]).

Theorem 5.2.1. *If there exists an $\epsilon > 0$ such that for every queue length vector $\mathbf{q}(t)$*

$$\frac{\tilde{f}(\mathbf{q}(t))}{f(\mathbf{q}(t))} \geq 1+\epsilon, \tag{5.3}$$

where the expectations are over the distributions of the arrival and channel state processes, then $\tilde{\Lambda} \supseteq (1+\epsilon)\Lambda$.

Based on this theorem, the idea of the proposed methods is to try to maximize the quantity given by (5.2), which is the negative part of the drift of the quadratic Lyapunov function for the system under a proposed scheme.

Since at most one user can be scheduled on a channel, $z_{kn}(t)=1$ only for the user with the maximum weight at channel n.

Unless stated otherwise, all expectations in the remainder of the paper are taken over the stationary distribution of the channel states and the decisions taken.

5.3 Randomized scheme

5.3.1 Algorithm Description

First we will examine the feedback scheme where a user whose rate supported by the channel is above the threshold feeds back at random. We will denote as $U_n(t)$ the number of users that feed back in channel n at timeslot t and $\mathcal{U}_n(t)$ the corresponding set. In addition, the set of users that have not fed back will be denoted as $\mathcal{N}_n(t)$, with $N_n(t)$ its cardinality. In detail, this scheme works as follows:

1. At the beginning of the slot, the base station broadcasts pilot signals (of duration that is assumed negligible).

2. The base station requests the user with maximum queue length, say user k^*, to report its channels. After this is done, it broadcasts the channel states at the corresponding channels. This implies that if $U_n(t)$ users in total (that is *including* the user with the maximum queue whose channel states have been requested by the base station) feed back on channel n, transmission will be done for the remaining duration of $(1-\beta(1+U_n(t)))T_s$

3. At each channel n, each user k compares its current achievable rate with the broadcasted channel state $r_{k^*n}(t)$. If $r_{kn}(t) < r_{k^*n}(t)$, the user does not report its channel state for channel n. Otherwise, he reports the channel state with some probability p.

4. At each channel n, as soon as users have finished reporting, the base station selects the user to schedule using a MaxWeight type of criterion, i.e. is scheduled the user that maximizes the quantity $q_k(t)(1 - \beta(1 + U_n(t)))r_{kn}(t)$.

The intuition of introducing this feedback probability in the scheme in [68] is that it can be tuned in a way so that fewer users will feed back while still scheduling good users for transmission on each channel. In the remainder of the paper, the proposed scheme will be denoted as "randomized". Also, we will refer to the quantity $q_i(t)r_{in}(t)$ as "weight" of user i in channel n.

5.3.2 Increasing the Stability Region with the randomized scheme

In this section we work on the case where there is enough time in the slot for each user to probe every channel, i.e. $(1-\beta K)>0$. Denoting

$$W_n(\mathbf{q}(t)) = \mathbb{E}\left\{\max_{k\in\{1,..,K\}}[q_k(t)r_{kn}(t)]\right\}$$

and $W(\mathbf{q}(t)) = \sum_{n=1}^{N} W_n(\mathbf{q}(t))$, we have:

Lemma 5.3.1. *For any vector of queue lengths, the following holds:*

$$\frac{\tilde{f}(\mathbf{q}(t))}{f(\mathbf{q}(t))} \geq \frac{1+r(\mathbf{q}(t),p)\epsilon}{1+\epsilon} \tag{5.4}$$

where $\epsilon>0$ is the increase of the stability region guaranteed by SSF with respect to full probing (i.e. a scheme where every user feeds back on every channel) and

$$r(\mathbf{q}(t),p) = (1-(K-2)S(\mathbf{q}(t)))\,p^2 + \frac{1-2\beta}{\beta}S(\mathbf{q}(t))p - \frac{1-\beta K}{\beta}S(\mathbf{q}(t)). \tag{5.5}$$

In the above,

$$S(\mathbf{q}(t)) = \frac{W(\mathbf{q}(t))}{\sum_{n=1}^{N}(\mathbb{E}(N_n(t)|\mathbf{q}(t))-1)W_n(\mathbf{q}(t))}. \tag{5.6}$$

Proof. We will denote the quantities corresponding to the full probing algorithm with a hat over the symbols. The schedules decided in SSF and full probing schemes (after feedback has been done) is the same, picking the user with the maximum product $r_{in}(t)q_i(t)$ in every channel n [68]. Note also that this value does not depend on the number of users probing each channel in the SSF algorithm, which implies that its expectation is independent of the expectation of the number of users probing. Then, we have $f(\mathbf{q}(t)) = \hat{f}(\mathbf{q}(t)) + \beta\sum_{n=1}^{N}(\mathbb{E}\{N_n(t)|\mathbf{q}(t)\}-1)W_n(\mathbf{q}(t))$, therefore

$$\frac{f(\mathbf{q}(t))}{\hat{f}(\mathbf{q}(t))} = 1 + \frac{\beta\sum_{n=1}^{N}(\mathbb{E}\{N_n(t)|\mathbf{q}(t)\}-1)W_n(\mathbf{q}(t))}{\hat{f}(\mathbf{q}(t))} := 1+\epsilon \tag{5.7}$$

with $\epsilon>0$.

Now we will do the same procedure for the quantities in the randomized scheme. Since now the user with the maximum weight is not guaranteed to feed back, we cannot proceed as above. However, a lower bound can be found considering the following: For every channel n, if the user with the maximum weight has probed then is scheduled, otherwise no user is scheduled. This is a lower bound since even if the user with the maximum weight is not probed there will be some other user with nonzero rate scheduled with some probability. Denoting $\tilde{\mathcal{U}}_n(t)$ the set of the users that have fed back at channel n at slot t and by $\tilde{\mathcal{N}}_n(t)$ the set of users that have not, we have

$$
\begin{aligned}
\tilde{f}(\mathbf{q}(t)) \geq \mathbb{E}&\left\{\sum_{n=1}^{N}(1-\beta(\tilde{U}_n(t)+1))\sum_{i\in\tilde{\mathcal{U}}_n(t)} q_i(t)r_{in}(t)z_{in}(t)|\mathbf{q}(t)\right\} \\
&= \sum_{n=1}^{N}\mathbb{E}\left\{(1-\beta(\tilde{U}_n(t)+1))\sum_{i=1}^{K} q_i(t)r_{in}(t)z_{in}(t)|\mathbf{q}(t)\right\} - \\
&\sum_{n=1}^{N}\mathbb{E}\left\{(1-\beta(\tilde{U}_n(t)+1))\sum_{i\in\tilde{\mathcal{N}}_n(t)} q_i(t)r_{in}(t)z_{in}(t)|\mathbf{q}(t)\right\} - \\
&\sum_{n=1}^{N}\mathbb{E}\left\{(1-\beta(\tilde{U}_n(t)+1))\sum_{i\in\tilde{\mathcal{N}}_n(t)} q_i(t)(t)r_{in}(t)z_{in}(t)|\mathbf{q}(t)\right\} \\
&\geq \hat{f}(\mathbf{q}(t)) + \beta\sum_{n=1}^{N}\left(\mathbb{E}\left\{\tilde{N}_n(t)|\mathbf{q}(t)\right\}-1\right)W_n(\mathbf{q}(t)) - \\
&\sum_{n=1}^{N}\mathbb{E}\left\{(1-\beta(\tilde{U}_n(t)+1))\sum_{i\in\tilde{\mathcal{N}}_n(t)} q_i(t)r_{in}(t)z_{in}(t)|\mathbf{q}(t)\right\}
\end{aligned}
\tag{5.8}
$$

To proceed further, we use that in the randomized scheme a user among the $U_n(t)-1$ (i.e. excluding the user polled by the base station) whose channel is better than the broadcasted feeds back with probability p independently of anything else. This implies that the average number of users that feed back after the threshold has been set will be $p\mathbb{E}\{U_n(t)-1|\mathbf{q}(t)\}$. So there is

$$
\mathbb{E}\left\{\tilde{U}_n(t)|\mathbf{q}(t)\right\} = 1 + p(K-1-\mathbb{E}\{N_n(t)|\mathbf{q}(t)\}). \tag{5.9}
$$

Now consider the third term in (5.8) and denote $\mathcal{X}_n$ the event that the user with the maximum queue is *not* the user with the maximum weight in channel n. This is the event for which the user with the maximum weight at channel n has a probability of not feeding back, thus the event under which the sum $\sum_{i\in\tilde{\mathcal{N}}_n(t)} q_i(t)r_{in}(t)z_{in}(t)|\mathbf{q}(t)$ can be nonzero. In this case, the probability that the sum over $i\in\tilde{\mathcal{N}}_n(t)$ being nonzero is $1-p$, since the user with the maximum weight will not feed back with this probability. Denote thus $\mathcal{X}'_n$ the event where the user with the maximum weight does not feed back. There is

$\mathbb{P}\{\mathcal{X}_n'|\mathcal{X}_n\} = 1-p$. Then we have

$$\sum_{n=1}^{N} \mathbb{E}\left\{(1-\beta(\tilde{U}_n(t)+1)) \sum_{i\in\tilde{\mathcal{N}}_n(t)} q_i(t)r_{in}(t)z_{in}(t)|\mathbf{q}(t)\right\} =$$
$$\sum_{n=1}^{N} \mathbb{P}(\mathcal{X}_n)\mathbb{P}(\mathcal{X}_n'|\mathcal{X}_n)\mathbb{E}\left\{\left(1-\beta(\tilde{U}_n(t)+1)\right) \sum_{i\in\tilde{\mathcal{N}}_n(t)} q_i(t)r_{in}(t)z_{in}(t)|\mathbf{q}(t),\mathcal{X}_n,\mathcal{X}_n'\right\}$$
$$= \sum_{n=1}^{N}(1-p)\mathbb{P}(\mathcal{X}_n)\left(1-\beta\left(\mathbb{E}\left\{\tilde{U}_n(t)\right\}+1\right)\right)W_n(\mathbf{q}(t))$$
$$\leq (1-p)\sum_{n=1}^{N}\left(1-\beta\left(\mathbb{E}\left\{\tilde{U}_n(t)\right\}+1\right)\right)W_n(\mathbf{q}(t)).$$

Here, we have used that the expectation is conditioned on the fact that the user with the maximum weight does not feed back, therefore is contained in the set $\tilde{M}_n(t)$ and that each user feeds back independently.

Therefore, applying the above in (5.8) and using (5.9), we obtain

$$\tilde{f}(\mathbf{q}(t)) \geq \hat{f}(\mathbf{q}(t)) + \left(\sum_{n=1}^{N}(\mathbb{E}\left\{N_n(t)|\mathbf{q}(t)\right\}-1)W_n(\mathbf{q}(t)) - (K-2)W(\mathbf{q}(t))\right)\beta p^2$$
$$+ p(1-2\beta)W(\mathbf{q}(t)) - (1-\beta K)W(\mathbf{q}(t)). \tag{5.10}$$

The stated result follows combining the above with (5.7). □

Using Lemma 5.3.1 and Theorem 5.2.1 we get the following:

Theorem 5.3.1. *If $r(\mathbf{q}(t),p) > 1, \forall \mathbf{q}(t)$ then the stability region is guaranteed to increase with respect to the SSF algorithm. Moreover, this guaranteed increase is the biggest for feedback probability*

$$p^* = \min\left\{1, \frac{1-2\beta}{2\beta}\frac{S(\mathbf{q}(t))}{(K-2)S(\mathbf{q}(t))-1}\right\}. \tag{5.11}$$

Proof. Assume that for every $\mathbf{q}(t)$, $r(\mathbf{q}(t),p) \geq 1+\delta(p) > 1$. Then, denoting $\epsilon' = \frac{\epsilon\delta(p)}{1+\epsilon}$, from Lemma 1 it follows that $\frac{\tilde{f}(\mathbf{q}(t))}{f(\mathbf{q}(t))} > 1+\epsilon'$, and using Theorem 5.2.1 we conclude that the stability region of the randomized scheme is at least $(1+\epsilon')$ times bigger the stability region of SSF. Also, note that the ratio (5.4) is increasing in $r(\mathbf{q}(t),p)$, which in turn is concave in p. Therefore optimizing over it we get the stated result. □

It is worth noting that optimizing according to the above result implies that $S(\mathbf{q}(t))$ is known at every time slot. This assumes that the probability distributions of the channels are known and requires some complex computations since this quantity $S(\mathbf{q}(t))$ should be frequently updated. Therefore, we will present in the next section a simple version of our algorithm that guarantees the expansion of the stability region in the worst case scenario. The interest in this case is that the probability p will be independent of $S(\mathbf{q}(t))$.

In the above analysis we have implicitly assumed that $\mathbb{E}\{N_n(t)|\mathbf{q}(t)\} > 1$ for each channel. Recall that K, being the number of users, can take only positive integer values. From [68] we have that for every channel, $\mathbb{E}\{N_n(t)\} \geq \frac{1}{2}(1+\frac{1}{L})(K-1)$. Therefore, the assumption holds for every $K \geq 3$, i.e. whenever there are at least three users in the system. This is the case where it actually makes sense to use this kind of schemes. Indeed, for the case where $K = 1$ there is essentially no scheduling problem. For $K = 2$, a scheme where every user feeds back is always better than the randomized one and SSF since both require a fraction of timeslot for the base station to broadcast the channel states of the user with maximum queue.

5.3.3 Approximate Randomized Scheme

In order to simplify the implementation of our feedback algorithm, we provide in this section an algorithm where the probability to feed back, p, does not depend on quantity $S(\mathbf{q}(t))$. For that, let us consider the case where the guaranteed region expansion (i.e. the lower bound of region increase given by Lemma 5.3.1 and Theorem 5.3.1) is minimal. We can show that this happens when all channel rates are uniformly distributed. This can be proven as follows. Let us denote by $N_{uni} := \mathbb{E}\left\{N_n(t)|\mathbf{q}(t), \text{uniform channel distribution}\right\} = (1/2 + 1/(2L))(K-1)$ (relation given in [68]). By (5.10), (5.11) and (5.6), it follows that the increase in the stability region *guaranteed* by the randomized scheme with respect to the full probing case is increasing as $\mathbb{E}N_n(t)|\mathbf{q}(t)$ increases. For each channel, we can prove that $N_{uni} \leq \mathbb{E}\left\{N_n(t)|\mathbf{q}(t)\right\}$ for any possible distribution of the channel states, in other words the case with uniform channel distribution has the worst lower bound gain with respect to the full probing case. The detailed proof of this result is not provided here since it can be obtained directly from the results in [68]. Therefore, examining this case gives a lower bound on the *guaranteed* achievable improvement with respect to full probing.

From the analysis in the previous subsection it follows:

Corollary 5.3.1. *When channel rates are uniformly distributed the feedback probability that maximizes the guaranteed enlargement of the stability region is given as* $p^*_{uni} = \min\left\{1, \frac{1-2\beta}{2\beta}\frac{2L}{2KL+K-3L-1}\right\}$.

Proof. Since the achievable rates are now identically distributed among subcarriers and users, it will also hold that $W_n(t) = W'(t)$, therefore $W(t) = \sum_{n=1}^{N} W_n(t) = NW'(t)$. So in the uniform distribution case we have $S(t) = \frac{NW'(t)}{W'(t)N(N_{uni}-1)} = \frac{1}{(K-1)\left(\frac{L+1}{2L}-1\right)}$. Replacing in (5.11) we get the stated result. $\square$

An attractive property of the feedback probability in this case is that it reduces the implementation complexity of the randomized version of SSF in practice. Therefore, and even if the distributions of the channel states are not homogeneous, we propose in this section to simplify the implementation by using p^*_{uni} given above instead of p^* given by (8). We call this algorithm "approximate randomized SSF". Also, note that for channel distribution other than the uniform there will be $p^* > p^*_{uni}$, which implies that a user implementing the approximate method will feed back on fewer channels. It is worth noting also that the uniform distribution represents the worst case in terms of guaranteed enlargement of the stability region given by the lower bound of $\tilde{f}(\mathbf{Q}(t))/\hat{f}(\mathbf{Q}(t))$. However, this lower bound may not be tight which means that the real increase of the stability region is higher than the lower bound developed here. In other words, using p^*_{uni} will not necessarily mean that the region expansion is minimum. In fact, p^*_{uni} ensures that we still have an improvement when the lower bound of the region increase is in its worst case (thus there is a guarantee of minimum region expansion).

5.3.4 Simulation Results and Discussion

In order to illustrate the gains and operation of the randomized feedback scheme we will consider a single cell downlink with $N = 15$ channels. The channels are assumed to be i.i.d. across users, frequencies, and time slots and the achievable rates (in bits per time slot) are as in Table 5.1 (the rates are calculated according to the LTE specifications, with $T_s = 1ms$).

Rate (bits/slot):	0	25	39	63	101	147	197	248
Probability:	0.03	0.04	0.05	0.05	0.06	0.06	0.09	0.09
Rate (bits/slot):	321	404	458	558	655	759	859	933
Probability:	0.1	0.1	0.09	0.06	0.06	0.05	0.04	0.03

Table 5.1: Achievable rates and probabilities used for the simulations

We set the traffic patterns to be i.i.d. Poisson, with the same arrival rate, λ bits per slot for each user. We run simulations lasting 10000 time slots each for different arrival rates and plot the average total queue length at each simulation for SSF, randomized SSF with probing probability as derived in Theorem 5.3.1

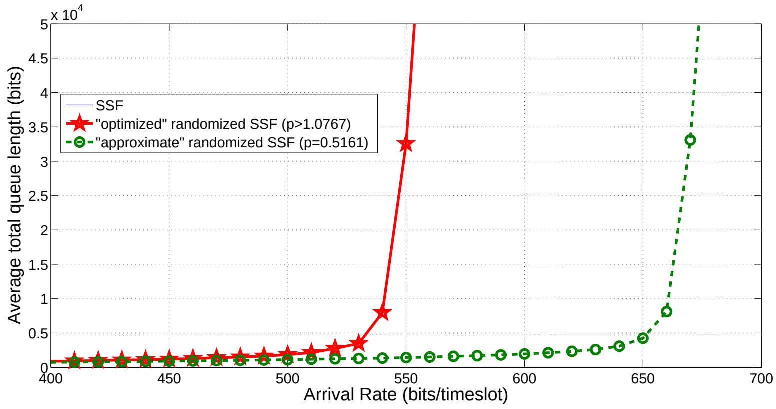

Figure 5.1: Average Total Queue Length for Different Mean Arrival Rates for 9 users and $\beta = 0.1$

(denoted "Optimized randomized SSF") and the approximate probability as set in Section 5.3.3.

At first we simulate the system with $\beta = 0.1$ and $K = 9$ users. In this case full probing is possible. The results are plotted in Fig. 5.1. We can see that the randomized version of the algorithm obtained via optimizing the upper bound is the same as SSF here, while the probability of probing in the approximate algorithm is smaller. Also, the performance of the approximate algorithm is better from the other two.

In Fig. 5.2 we present the results of a scenario with $K = 25$ users and two different values of β, namely 0.05 and 0.01. Note that in both of these cases full probing is not possible.

Again, the approximate version of the algorithm results in a lower probability to feed back than the version that optimizes the upper bound and performs better. In turn, the latter version performs better than SSF. Also, from Figures 5.1 and 5.2 we can see that the stability region of the system shrinks under all algorithms as β and/or the number of users K grow larger. In the case of SSF this happens because as these parameters grow larger, more time needs to be devoted to channel feedback, leaving fewer time for transmission. However, in the randomized versions the main reason for the rate decrease is that it becomes more possible that the user with maximum weight will not feed back his channel state and subsequently another user will be scheduled instead. As we can see in the figures, the decrease in the stability region is slower in the case of the randomized algorithms. This demonstrates that there is a gain with respect to SSF algorithm and moreover that the relative gains of randomizing the SSF

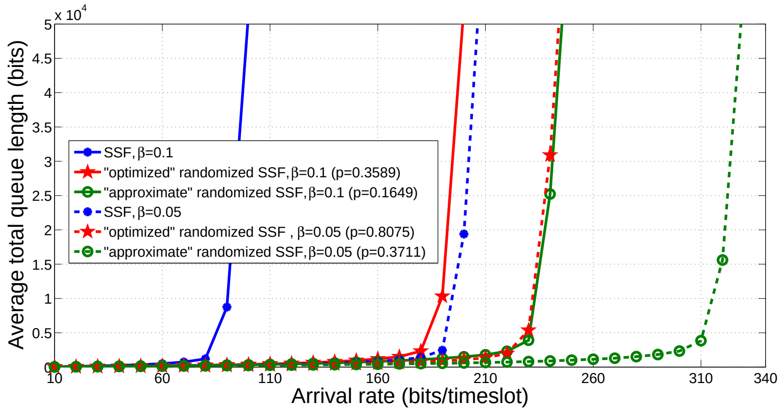

Figure 5.2: Average Total Queue Length for Different Mean Arrival Rates for 25 users and different values of β

algorithm are bigger when there are more users and/or channel probing is more costly.

The main reason why the approximate algorithm outperforms the one optimizing the probability so that the increase guaranteed by Theorem 5.3.1 is that the bound to which the optimized probability corresponds to is not tight. In fact, the theoretical analysis in this paper has been done in terms of region increase guarantee and this has been studied using the lower bounds developed in the previous sections. These bounds are not necessarily tight which means that the real expansion is higher than the lower bound. Recall that in the course of derivation of equation (5.5), the quantity was bounded assuming implicitly that (i) if the user with the maximum weight has not probed a channel then no user is scheduled in the channel and (ii) the user polled by the base station is never the user with the maximum weight. As seen previously the approximate probability p^*_{uni} is less or equal than the one obtained through full optimization.

5.4 Scheme based on stopping

The randomized scheme discussed in Section 5.3 does provide an increase in the stability region of the system with respect to the SSF algorithm, however its main drawback is that the derivation of the probability with which a user should feed back is based on a loose upper bound on this increase. In addition, this probability depends in general on the channel statistics and queue lenghts every slot (the approximate version seems to yield good performance but it is still an approximation). In this Section we propose an alternative scheme, again based on the SSF rule and with no need of the channel statistics, where the base station decides when to stop the feedback procedure and schedule a user, in an attempt to make the quantity of (5.2) as big as possible.

5.4.1 Description of the scheme

We assume that at the beginning of each timeslot the base station broadcasts a pilot signal of negligible duration, so that the users can know their current channel states. Denote $\mathcal{U}_n(m,t)$ the set of users that have fed back at control slot m of timeslot t. The proposed algorithm actually considers every channel in isolation and consists in the following steps for every time slot t at each channel n:

1. The base station requests the CQI of the user with the biggest queue length, k^*.

2. If after receiving feedback in the first minislot it holds that
$$\frac{1-\beta}{1-3\beta}\frac{q_{k^*}(t)}{q_k(t)}r_{k^*n}(t) \geq R_L, \forall k \neq k^*,$$
then the base station transmits at user k^* at its achievable rate for the rest of the timeslot (and so the algorithm terminates). Otherwise, it broadcasts $r_{thr,n}(t) := r_{k^*n}(t)$ during the second feedback minislot.

3. For each minislot $m > 2$, at the beginning the base station chooses $k^* = \arg\max_{k\in\mathcal{U}_n(m,t)}\{r_{kn}(t)q_k(t)\}$. If it holds that
$$\frac{1-m\beta}{1-(m+1)\beta}\frac{q_{k^*}(t)}{q_k(t)}r_{k^*n}(t) \geq R_L, \forall k \notin \mathcal{U}_n(m,t), \tag{5.12}$$
then the base station transmits to user $r^*$and the algorithm terminates. Otherwise, user $i \notin \mathcal{U}_n(m,t)$ with rate $r_{in}(t) > r_{thr,n}(t)$ feeds back according to a decentralized rule.

4. The algorithm stops when there is no user to feed back on channel n (that is all remaining users not yet fed back have worse channel state than the one broadcasted) or when $m = \lfloor\frac{1}{\beta}\rfloor$. The latter means that this is the last minislot; in this case, the base station transmits to the user with the maximum weight among the ones that fed back.

The main idea behind the algorithm is to transmit when there is no possibility that receiving further feedback will increase the weight of the user scheduled in the channel, thus increasing (5.2). For example, if the user with the maximum queue length has the maximum possible rate allowed by the standard on channel n, then it is useless to continue receiving feedback from other users. Formally we can show the following:

Proposition 5.4.1. *Under the algorithm described in this section, $\tilde{\Lambda} \supset \Lambda$, where Λ is the stability region when the algorithm in [68] is used.*

Proof. We consider the beginning of minislot m at channel n at timeslot t and denote $k_n^*(m,t) = \text{argmax}_{k \in \mathcal{U}_n(m,t)} \{r_{kn}(t)q_k(t)\}$, that is the user with the maximum weight at this channel so far. If $m > 2$, then if user $i \notin \mathcal{U}_n(m,t)$ feeds back, the maximum weight of the channel in minislot $m+1$ will increase if $(1+\beta(m+1))r_{in}(t)q_i(t) > (1-\beta m)\max_{k \in \mathcal{U}_n(m,t)}\{r_{kn}(t)q_k(t)\}$. This implies that, since the queue lengths vector is known to the base station, the weight in this channel gets bigger if $i \notin \mathcal{U}_n(m,t)$ feeds back at minislot m if

$$r_{in}(t) > \hat{r}_{in}(m,t) := \frac{1-\beta m}{1-\beta(m+1)} \frac{\max_{k \in \mathcal{U}_n(m,t)}\{r_{kn}(t)q_k(t)\}}{q_i(t)}. \qquad (5.13)$$

Consider now the case where

$$\hat{r}_{in}(m,t) \geq R_L, \forall i \notin \mathcal{U}_n(m,t). \qquad (5.14)$$

Since $\frac{1-\beta m}{1-\beta(m+1)}$ is increasing in m, $\hat{r}_{in}(m+1,t) > \hat{r}_{in}(m,t) > R_L, \forall i \notin \mathcal{U}_n(m,t)$. This analysis implies that if (5.14) holds in the beginning of minislot m then the weight of the user scheduled at channel n will not increase any further. Similar analysis holds for $m = 1$ as well, taking though into account that if the base station decides not to transmit and at least one user is above the threshold, then it can transmit again after minislot $m = 3$ the earliest due to the second minislot used for broadcasting (thus the denominator of Step 2 in the algorithm).

The above implies that, given any (possibly randomized) rule for the decentralized feedback scheme, for any realization of this rule under any realization of the channel states and any fixed queue length vector we have that if $r_{k^*(n)n(t)} < R_L$, it holds

$$\left[1-\beta\tilde{M}_n(t)\right]^+ \sum_{i=1}^{K} q_i(t)r_{in}(t)\tilde{z}_{in} \geq [1-\beta M_n(t)]^+ \sum_{i=1}^{K} q_i(t)r_{in}(t)z_{in} \qquad (5.15)$$

with probability 1. In the case where the $r_{k^*(n)n(t)} = R_L$, the user with the maximum queue length is the user with the maximum weight already. This user is scheduled right after the first minislot in our algorithm while under SSF the second minislot is also used for the broadcasting of this rate, so the weight under our algorithm in this case is stricly bigger than SSF with probability one.

This analysis implies that for every channel $n = 1, ..., N$

$$\begin{aligned} \mathbb{E}\left\{\left[1-\beta\tilde{M}_n(t)\right]^+ \sum_{i=1}^{K} q_i(t) r_{in}(t) \tilde{z}_{in} \middle| \mathbf{q}(t)\right\} > \\ \mathbb{E}\left\{[1-\beta M_n(t)]^+ \sum_{i=1}^{K} q_i(t) r_{in}(t) z_{in} \middle| \mathbf{q}(t)\right\}. \end{aligned} \tag{5.16}$$

Summing over all channels and using the fact that they are independent we get $\tilde{f}(\mathbf{q}(t)) > f(\mathbf{q}(t))$, and combining this with Theorem 5.2.1 completes the proof. □

A further issue is how exactly the users that have better rate than the broadcasted one can be coordinated to feed back. This can be done for example if the base station ranks the users and communicates this ranking with them (e.g. it can be a ranking according to their IDs communicated at the beginning of the systems' operation reflecting a priority of each user), and divides the portion of the second minislot that remains after the threshold broadcast among the users (in a TDMA manner in each channel). In this case, when it is the turn of each user, they can send a signal if their rate at the channel is above the threshold and send nothing otherwise. In any case, the number of minislots used for the feedback phase for the SSF algorithm does not depend on the way the users above the threshold feed back, our algorithm outperforms SSF under any user ordering scheme. However, the actual ordering scheme *will* affect the stability region of the stopping-based algorithm. For the threshold broadcast step to make sense, we must have $\beta < 1/3$ and $K > 2$ users. However, note our result holds for β and K having any value respecting the mentioned conditions, so there may be $\beta K > 1$, case in which there is not enough time for every user to feed back in every channel.

5.4.2 Analysis of the time spent for feedback for i.i.d. channels

Here we will provide some mathematical analysis on the number of minislots taken up by our proposed policy. In order to simplify the model, we will assume that all channels are identically distributed with $\mathbb{P}\{r_{kn}(t) = R_l\} = q_l$. In addition, we will assume that the users feed back according to a ranking based on the queue lengths. This can be implemented as follows: The base station can broadcast a ranking of the users according to the queue lengths with the user with the highest queue length first at the beginning of the timeslot (e.g.at the beginning of the first minislot and then in the remaining time of this minislot the first user in the ranking feeds back in all channels). Then, the procedure described in the previous subsection for determining the sequence at which users will feed back is followed. Note that, since we are actually interested in maximizing the quantity (5.2) and the channels are i.i.d. , this method will give the biggest stability region (biggest value of (5.2)) over any feedback sequence under the proposed scheme.

Given the stopping condition at each minislot and the above mentioned feedback scheme, we can further see that the expected number of minislots used is the biggest when the queue lengths are equal since it leads to $\frac{q_{k^*}(t)}{q_k(t)} = 1$ in the stopping condition. Therefore we will examine this setting in order to obtain a worst case analysis of the scheme. Since all queues are equal, without loss of generality, ranking will be assumed to be according to the user IDs in ascending order (i.e. user 1 feeds back first etc.). Denote $\tilde{p}_n(m)$ the probability that exactly m minislots are used at carrier n under our scheme and $p_n(m)$ the corresponding quantity for the SSF algorithm. Note that when $m > \lfloor \frac{1}{\beta} \rfloor$, more time than the duration of the timeslot needs to be used. So, eventually the base station does not transmit at all in the slot (this goes for the SSF algorithm as our proposed one stops at most after the minislot just before the last that can fit in the timeslot duration). Also denote $F_l = \mathbb{P}\{R_{kn} \geq r_l\} = \sum_{l=l}^{L} q_l$, so $F_{L+1} = 0$. For the SSF algorithm, the number of minislots needed is the number of users out of the remaining $K-1$ that have rates over the threshold plus the two minislots in the beginning, so we have for

$$p_n(m) = \sum_{l=1}^{L} q_l \binom{K-1}{m-2} F_{l+1}^{m-2} (1 - F_{l+1})^{K-m+1}, m \geq 2$$

and zero for m=1.

For the proposed scheme, note that we have for $m = 1$

$$\tilde{p}_n(1) = \sum_{l=1}^{L} q_l I_{\{r_l \geq \frac{1-3\beta}{1-\beta} r_L\}}.$$

For $m = 2$, it corresponds to the case when the stopping condition is not fulfilled after the user with the maximum queue lengths feeds back but no user among the remaining $K-1$ has greater rate, thus we have

$$\tilde{p}_n(2) = \sum_{l=1}^{L} q_l I_{\{r_l < \frac{1-3\beta}{1-\beta} r_L\}} (1 - F_{l+1})^{K-1}.$$

For $m = 3$, the corresponding event, given the threshold rate (the rate of the user with the maximum queue length) is that the stopping condition did not hold for the first minislot and that either only one of the $K-1$ users has rate above the threshold or this happens for more than one user but the stopping condition holds after a user feeds back in this minislot. Replacing the probabilities we get

$$\tilde{p}_n(3) = \sum_{l=1}^{L} q_l I_{\{r_l < \frac{1-3\beta}{1-\beta} r_L\}}$$
$$\Bigg((K-1)F_{l+1}(1-F_{l+1})^{K-2} + \big(1-(K-1)F_{l+1}(1-F_{l+1})^{K-2} - (1-F_{l+1})^{K-1}\big)$$
$$\sum_{l'=l+1}^{L} q_{l'} I_{\{r_{l'} < \frac{1-4\beta}{1-3\beta} r_L\}} \Bigg).$$

For $m > 3$, getting closed form expressions like the above becomes more difficult, since for every m this probability depends on which users have fed back and their channel realizations, thus boiling down to a combinatorial problem. Define for $m > 2$ the outcome of the feedback process until and including the m-th minislot as

$$\pi(m) = \Bigg((i_1(\pi), R_{i_1(\pi)n}(t)), (0,0), (i_3(\pi), R_{i_3(\pi)n}(t)), ..., (i_m(\pi), i_m(\pi)(t)) \Bigg).$$

More specifically $i_j(\pi)$ is the user that fed back at minislot $j \leq m$ and $R_{i_j(\pi)n}(t)$ the corresponding achievable rate. A realization
$\pi(m) = ((i_1, r^{(1)}), (0,0), ..., (i_j, r^{(j)}, .., (i_m, r^{(m)}))$ is possible if the following conditions are met: (i)$r^{(j)} > r^{(1)}, \forall j = 3, .., m$, (ii)$(1-\beta)r^{(1)} < (1-3\beta)r_L$ (iii)$(1-\beta j)\max\{r^{(1)}, r^{(3)}, ..., r^{(j-1)}, r^{(j)}\} < (1-\beta(1+j))r_L, \forall j = 1, .., m-1$ and (iv) either $m-1$ exactly user have rates above the threshold or $(1-\beta m)\max\{r^{(1)}, r^{(3)}, ..., r^{(m-1)}, r^{(m)}\} \geq (1-\beta(1+m))r_L$. Let $\Pi(m, (1, r_l))$ be the set containing all possible realizations of the feedback algorithm lasting exactly m minislots when the user requested to feed back first has rate r_l. Then we have that for $m > 2$:

$$\tilde{p}_n(m) = \sum_{l=1}^{L} q_l \sum_{\pi \in \Pi(m,(1,r_l))} \prod_{j=3}^{m} \mathbb{P}\{R_{i_j(\pi)n}(t) = r^{(j)}(\pi)\}(1-F_{l+1})^{i_j(\pi)-i_{j-1}(\pi)}.$$

The above equation comes from the fact that since users are ranked using their IDs, if after user i_{j-1}, user i_j feeds back, it implies that the users with IDs from $i_{j-1}+1$ till and including i_j-1 have achievable rates below the broadcasted threshold at channel n.

The results in this subsection can be used to numerically obtain an estimate of the mean amount of time needed in each timeslot for the feedback procedure to be executed under our algorithm.

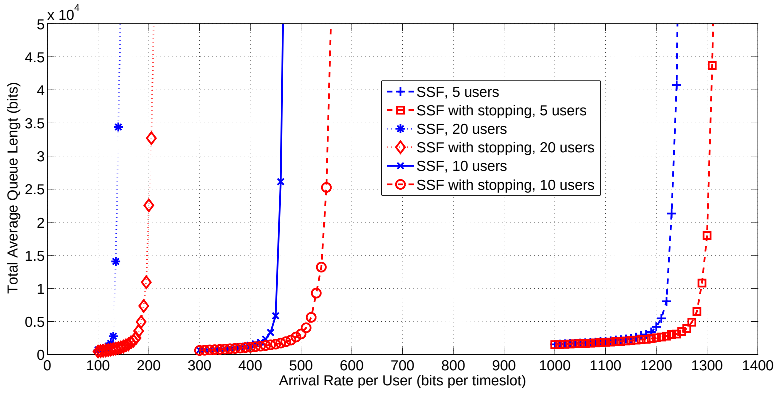

Figure 5.3: Average Total Queue Length for Different Mean Arrival Rates for $\beta = 0.1$ and different number of users

5.4.3 Simulation Results

In order to illustrate the gains from our proposed feedback and scheduling algorithm, we will consider for convenience a downlink system with $N = 15$ channels which identically distributed among them and among users, and i.i.d. in time. The possible rates and corresponding probability distributions are the ones shown in Table 5.1. In addition, the traffic processes are Poisson with the same rate for each user and i.i.d. in time. What we are showing here, therefore, is stability behaviour on the line $\lambda_1 = \lambda_2 = ... = \lambda_K$ in a system with identical channels for each user. The point where the system is becoming unstable is the point where the total average queue length plotted in the figures that follow starts increasing very steeply. We are comparing the performance of our algorithm with the one in [68] applied directly in multichannel systems.

In Fig. 5.3 we present the simulation results for different numbers of users and $\beta = 0.1$. As the number of users grows, the stability region of both algorithms shrinks, and the region under our algorithm is bigger than the region under [68]. However, we can observe that the absolute difference between the two algorithms is very similar for each of the cases shown, which suggests that the absolute difference in the stability regions between the two algorithms does not change much with the number of users. An explanation for this is that the proposed stopping rule does not take at all into account the number of users, so (unless the number of users is so large that there is not enough time for everyone to feed back even in the SSF scheme) the degradation on the stability region of both algorithms is similar.

In Fig. 5.4, we present the results for different values of the fraction of time,

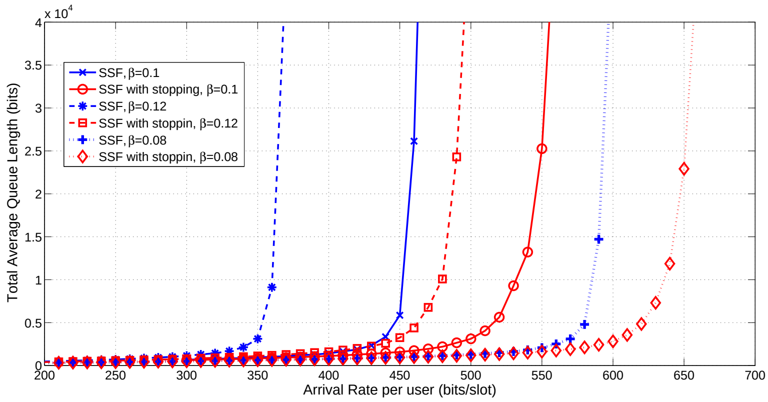

Figure 5.4: Average Total Queue Length for Different Mean Arrival Rates for 10 users and different values of β

β for one user to feed back on a channel in a system with 10 users.

Unlike the previous case, we observe that relatively small changes to the parameter β result to different absolute differences, and more precisely the bigger this parameter is, the bigger is the gain, with respect to the algorithm in [68], of using the proposed algorithm (and again, the stability region of both algorithms shrinks as β increases).

5.5 Conclusions

In this chapter we presented two feedback and scheduling algorithms for enlarging the stability region in multichannel systems when a fraction of the timeslot must be taken for each user to feed back. These algorithms extend the main idea of [68] where the achievable rate of the user with the maximum queue is set as the threshold for other users to feed back. We showed that properly randomizing the feedback decision and/or having the base station stop the feedback process can enlarge the stability region achieved by the system. Especially in the case of the scheme based on stopping, no need to know the statistics of the channel and arrival processes is required, and the scheme can be implemented even in cases with correlated arrivals/channels. This is a very desirable property for practical systems.

Chapter 6

Conclusions & Perspectives

This thesis addressed problems of allocation and control of radio resources in wireless systems, taking into account that each user requests a dynamic traffic process. The two main aspects studied were power/precoding control for the interference channel such that the probability of the queue of each user exceeding a threshold is at a desired value and joint scheduling/user selection and feedback in TDD single cell downlink systems (i.e. broadcast channels with feedback in TDD) in order to enlarge the stability region of the system. In this Chapter, we summarize the main results and illustrate possible extensions.

6.1 Conclusions and Extensions of the Thesis Results

As far as the interference channel was concerned, we have demonstrated that the heavy traffic approximation can be a tool to obtain useful guidelines to design power and precoding control algorithms. An immediate extension of the results of Chapter 3 is the case where the channel states are received with errors at each transmitter: if the statistics of the errors are known, then we can arrive at a similar model where the averaging will also be given over the channel estimation processes (this may require more cumbersome calculations in the system model though). In addition, the case where tranmitter serves multiple receivers in a TDMA manner, where the sequence of which user is served in each slot is pre-specified and remains constant (or changes very slowly) during the system's operation can be treated by slightly modifying our model. However, further extending the results using opportunistic and/or queue-based scheduling is not straightforward and needs more elaborate analysis. Another possible extension would be to find the beamforming control that minimizes the power in order to achieve the queue overflow constraints. The course of action here would be to (i) find the equilibrium allocation with the minimum average power and then (ii) for the resulting SDE , solve the problem trying to minimize the reserve power while satisfying the constraints. Theorem (3.3.1) holds also in this case but with more complicated expressions and with a resulting control problem that may be solvable only numerically, as decoupling of the multidimensional SDE will not be possible. Finally, one could use buffers in the receivers instead of the transmitters, in order to model video streaming, and use a similar asymptotic approach for beamforming control. A recent work [69] is in this spirit.

The main conclusion drawn from Chapters 4 and 5 is that the fact that the users can know their instantaneous channel state should be leveraged. More specifically, we have shown in Chapter 4 that, assuming idealized contention procedures, using a policy where the base station signals the number of users to get selected and the users decide in a contention-based decentralized manner who is going to participate in the schedule can enlarge the stability region of a MISO system with respect to centralized user selection. In addition, for a single antenna at the transmitter, a contention-based decentralized policy can achieve a big fraction of the ideal stability region of the system. Even in cases where minislots are to be used for control, users' knowledge of their channel states can be leveraged via a proper threshold-based schemes. Apart from the interpretation that the system can support more non-real time traffic demands, extension of the stability region implies achieving higher utility if a utility-based optimization approach is to be used. More specifically, define a utility function

$$U(\bar{\mathbf{r}}) = \sum_{k=1}^{K} U_k(\bar{r}_k),$$

where $\bar{r}_k$ is the long term average of the rate user k gets and $U_k(x)$ is a concave function. In this case, it can be shown that the optimal long term rate vector lies in the boundary of the stability region, see for example [15], therefore enlarging the stability region leads to higher optimal utility. In addition, at least for the setting of a single cell downlink system, a general method of utility optimization consists in creating virtual queues $q_k(t)$ with controlled arrival processes such that

$$\mathbf{a}(t) = \arg \max_{\mathbf{x}\in[0,A]^K} \left\{ VU(\mathbf{x}) - \sum_{k=1}^{K} x_k q_k(t) \right\},$$

with $V > 0$ a constant. The scheduling should be such that these virtual queues are stable. one can then show that the achieved utility is within $O(1/V)$ from the optimal, with the mean sum of queue lengths growing as $O(V)$ [15]. This general theory has been applied in scheduling and resource allocation in wireless networks with promising results, see for example [70], [16], [17]. With respect to our results, we can apply the same techniques proposed in Chapters 4 and 5 using the virtual queues. In addition, on Chapter 4 one can use the general network optimization framework of [15] and consider the aspect of energy consumed by the users for uplink training, and minimize it subject to stability of the system or examine the stability/utility maximization problem under constraints on the average power spent for training. A motivation for this kind of considerations is to prolong the users' battery life. Since the channel state knowledge becomes even more important in that case, our intuition is that decentralized user selection policies will be have even better performance than the centralized ones. Extension of the results in the thesis along these lines is the subject of ongoing work.

6.2 Future Work

The results of the present thesis concern basically the behaviour of the queue lengths in the system. However, a more important metric in practice is the delay/waiting time experienced by the data in the queues until they get transmitted. While delays and queue lengths are related quantities (for example the average delay is equal to the average queue length divided by the arrival rate by Little's law and intuitively we can expect that packets in a long queue will experience big delays), exact delay analysis is known to be much harder than queue length analysis. It would be interesting to see how delay-based scheduling algorithms perform, for example in the spirit of the recent work [71]. On the other hand, we can look in the large deviations regime in order to obtain additional information about the behaviour of the system. This kind of asymptotics have been also widely used in analysis of scheduling algorithms in queueing systems in general and wireless systems in particular (see e.g. [72, 73] for the latter), and recent works [74] and [63] study the performance of scheduling and feedback algorithms in this regime. Moreover, combining with the theory of effective bandwidth and effective capacity [75] to get some estimation of the tail of the delay distributions.

The approaches above and the schemes proposed in this thesis have the constraint that data are not dropped in the queues. On the other hand, most applications can tolerate a certain loss of data but have hard delay requirements. Recent works on this topic include [76], [77] for wireless networks with relatively simple physical layer and [78] for wired networks. It would be interesting to examine problems of these kind in the settings of small cell networks and/or massive MIMO systems. In the former, looking into theories like machine learning or decentralized/networked control systems may be essential in order to overcome the limited control backhaul connecting the cells. In the latter, since there will be many users in the cell, user selection is an important problem (the timeslot durations and coherence times are given) and it will be interesting to see if we can leverage our results to deal in the delay-sensitive case.

Finally, an aspect not taken into account in this thesis is the fact that, in a network, users arrive and depart, see e.g. [79] and references therein. For example, a user can arrive in the network, request a file and depart when this transfer has been completed. Scheduling and resource allocation in this setting posed more challenges, since MaxWeight is no longer throughput optimal [80]. This setting can be applied to provide robust performance evaluation of the recent approaches in proactive scheduling [81] and caching in the small cell base stations [82], proposed in order to make more efficient use of the limited backhaul. In addition these flow level models can be used to provide guidances on what files to prefetch, where and when.

Bibliography

[1] J. Hoydis, M. Kobayashi, and M. Debbah, "Green Small-Cell Networks," *IEEE Vehicular Technology Magazine*, vol. 6, no. 1, pp. 37 –43, Mar. 2011.

[2] M. Karakayali, G. Foschini, and R. Valenzuela, "Network coordination for spectrally efficient communications in cellular systems," *IEEE Transactions on Wireless Communications*, vol. 13, no. 4, pp. 56 –61, Aug. 2006.

[3] D. Gesbert, S. Hanly, H. Huang, S. Shamai Shitz, O. Simeone, and W. Yu, "Multi-Cell MIMO cooperative networks: A new look at interference," *IEEE Journal on Selected Areas in Communications*, vol. 28, no. 9, pp. 1380 –1408, Dec. 2010.

[4] G. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, no. 3, pp. 311–335, 1998.

[5] D. Gesbert, M. Kountouris, R. Heath, C.-B. Chae, and T. Salzer, "Shifting the MIMO paradigm," *IEEE Signal Processing Magazine*, vol. 24, no. 5, pp. 36–46, 2007.

[6] T. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Transactions on Wireless Communications*, vol. 9, no. 11, pp. 3590–3600, Sep. 2010.

[7] F. Rusek, D. Persson, B. K. Lau, E. Larsson, T. Marzetta, O. Edfors, and F. Tufvesson, "Scaling Up MIMO: Opportunities and Challenges with Very Large Arrays," *IEEE Signal Processing Magazine*, vol. 30, no. 1, pp. 40–60, 2013.

[8] R. Knopp and P. Humblet, "Information capacity and power control in single-cell multiuser communications," in *IEEE International Conference on Communications (ICC)*, vol. 1, Seattle, USA, 1995, pp. 331–335 vol.1.

[9] P. Viswanath, D. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1277–1294, 2002.

[10] P. Bender, P. Black, M. Grob, R. Padovani, N. Sindhushyana, and S. Viterbi, "CDMA/HDR: a bandwidth efficient high speed wireless data service for nomadic users," *IEEE Communications Magazine*, vol. 38, no. 7, pp. 70 –77, Jul. 2000.

[11] S. Sesia, I. Toufik, and M. Baker, *LTE, The UMTS Long Term Evolution: From Theory to Practice.* Wiley, 2009.

[12] M. Andrews, K. Kumaran, K. Ramanan, A. Stolyar, P. Whiting, and R. Vijayakumar, "Providing quality of service over a shared wireless link," *IEEE Communications Magazine*, vol. 39, no. 2, pp. 150 –154, Feb. 2001.

[13] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *IEEE Transactions on Automatic Control*, vol. 37, no. 12, pp. 1936 –1948, Dec. 1992.

[14] M. Neely, E. Modiano, and C. P. Li, "Fairness and optimal stochastic control for heterogeneous networks," *IEEE/ACM Transactions on Networking*, vol. 16, no. 2, pp. 396–409, Apr. 2008.

[15] M. J. Neely, "Stochastic network optimization with application to communication and queueing systems," *Synthesis Lectures on Communication Networks*, vol. 3, no. 1, pp. 1–211, 2010.

[16] D. Bethanabhotla, G. Caire, and M. Neely, "Utility optimal scheduling and admission control for adaptive video streaming in small cell networks," in *IEEE International Symposeum on Information Theory (ISIT)*, Istabul, Turkey, July 2013, pp. 1944–1948.

[17] D. Bethanabhotla, G. Caire, and M. J. Neely, "Adaptive Video Streaming in MU-MIMO Networks," *ArXiv e-prints*, Jan. 2014.

[18] M. Chiang, P. Hande, T. Lan, and C. W. Tan, "Power control in wireless cellular networks," *Foundations and Trends in Networking*, vol. 2, pp. 381–533, Apr. 2008.

[19] H. Dahrouj and W. Yu, "Coordinated beamforming for the multicell multi-antenna wireless system," *IEEE Transactions on Wireless Communications*, vol. 9, no. 5, pp. 1748 –1759, May 2010.

[20] G. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Transactions on Vehicular Technology*, vol. 42, no. 4, pp. 641 –646, Nov. 1993.

[21] S. Lakshminarayana, M. Assaad, and M. Debbah, "H-infinity control based scheduler for the deployment of small cell networks," in *Intl. Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks*, Princeton, NJ , USA, May 2011, pp. 9 –16.

[22] Y. Huang and S. Mao, "Downlink power control for variable bit rate videos over multicell wireless networks," in *IEEE International Conference on Computer Communications (IEEE INFOCOM)*, Shanghai, China, Apr. 2011, pp. 2561 –2569.

[23] H. Huang and V. Lau, "Delay-optimal user scheduling and inter-cell interference management in cellular network via distributive stochastic learning," *IEEE Transactions on Wireless Communications*, vol. 9, no. 12, pp. 3790 –3797, Dec. 2010.

[24] Y. Cui, V. Lau, R. Wang, H. Huang, and S. Zhang, "A Survey on Delay-Aware Resource Control for Wireless Systems: Large Deviation Theory, Stochastic Lyapunov Drift, and Distributed Stochastic Learning," *IEEE Transactions on Information Theory*, vol. 58, no. 3, pp. 1677 –1701, Mar. 2012.

[25] L. Georgiadis, M. J. Neely, and L. Tassiulas, "Resource allocation and cross-layer control in wireless networks," *Foundations and Trends in Networking*, pp. 1–144, Apr 2006.

[26] H. J. Kushner, *Heavy Traffic Analysis of Controlled Queuing and Communication Networks.* Berlin and New York: Springrer-Verlag, 2001.

[27] A. L. Stolyar, "Maxweight Scheduling in a Generalized Switch: State Space Collapse and Workload Minimization in Heavy Traffic," *The Annals of Applied Probability*, vol. 14, no. 1, pp. 1–53, 2004.

[28] S. Shakkottai, R. Srikant, and A. L. Stolyar, "Pathwise optimality of the exponential scheduling rule for wireless channels," *Advances in Applied Probability*, vol. 36, no. 4, pp. 1021–1045, 2004.

[29] S. Bhardwaj, R. J. Williams, and A. S. Acampora, "On the performance of a two-user MIMO downlink system in heavy traffic," *IEEE Transactions on Information Theory*, vol. 53, no. 5, pp. 1851 –1859, May 2007.

[30] S. Bhardwaj and R. Williams, "Diffusion approximation for a heavily loaded multi-user wireless communication system with cooperation," *Queuing Systems*, vol. 62, pp. 345–382, 2009.

[31] W. Dai, "Optimal Rate Scheduling via Utility-Maximization for J-User MIMO Markov Fading Wireless Channels with Cooperation," *ArXiv e-prints*, Jun. 2011.

[32] R. Buche and H. J. Kushner, "Control of mobile communications with time-varying channels in heavy traffic," *IEEE Transactions on Automatic Control*, vol. 47, no. 6, pp. 992 –1003, Jun. 2002.

[33] R. T. Buche and C. Lin, "Heavy traffic control policies for wireless systems with time-varying channels," in *Americal Control Conference (ACC)*, Jun. 2005, pp. 3972 – 3974.

[34] W. Wu, A. Arapostathis, and S. Shakkottai, "Optimal power allocation for a time-varying wireless channel under heavy-traffic approximation," *IEEE Transactions on Automatic Control*, vol. 51, no. 4, pp. 580 – 594, Apr. 2006.

[35] R. Berry and R. Gallager, "Communication Over Fading Channels with Delay Constraints," *IEEE Transactions on Information Theory*, vol. 48, no. 5, pp. 1135–1149, May 2002.

[36] H. Weingarten, Y. Steinberg, and S. Shamai, "The Capacity Region of the Gaussian Multiple-Input Multiple-Output Broadcast Channel," *IIEEE Transactions on Information Theory,*, vol. 52, no. 9, pp. 3936–3964, Sept. 2006.

[37] D. Love, R. Heath, V. Lau, D. Gesbert, B. Rao, and M. Andrews, "An overview of limited feedback in wireless communication systems," *IEEE Journal on Selected Areas on Communications*, vol. 26, no. 8, pp. 1341 – 1365, Oct. 2008.

[38] G. Caire, N. Jindal, M. Kobayashi, and N. Ravindran, "Multiuser mimo achievable rates with downlink training and channel state feedback," *IEEE Transactions on Information Theory*, vol. 56, no. 6, Jun. 2010.

[39] M. Kobayashi, N. Jindal, and G. Caire, "Training and Feedback Optimization for Multiuser MIMO Downlink," *IEEE Transactions on Communications*, vol. 59, no. 8, pp. 2228–2240, Aug.t 2011.

[40] L. Song, Z. Han, Z. Zhang, and B. Jiao, "Non-cooperative feedback-rate control game for channel state information in wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 1, pp. 188 –197, Jan. 2012.

[41] H. Boche and M. Wiczanowski, "The interplay of link layer and physical layer under MIMO enhancement: benefits and challenges," *IEEE Wireless Communications*, vol. 13, no. 4, pp. 48–55, Aug. 2006.

[42] ——, "Optimization-Theoretic Analysis of Stability-Optimal Transmission Policy for Multiple-Antenna Multiple-Access Channel," *IEEE Transactions on Signal Processing*, vol. 55, no. 6, pp. 2688–2702, Jun. 2007.

[43] C. Swannack, E. Uysal-Biyikoglu, and G. Wornell, "Low Complexity Multiuser Scheduling for Maximizing Throughput in the MIMO Broadcast Channel," in *Allerton Conference on Communications, Control, and Computing*, Monticello, IL, USA, 2004.

[44] M. Kobayashi and G. Caire, "An iterative water-filling algorithm for maximum weighted sum-rate of Gaussian MIMO-BC," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 8, pp. 1640–1646, Aug. 2006.

[45] C. Wang and R. Murch, "Optimal downlink multi-user MIMO cross-layer scheduling using HOL packet waiting time," *IEEE Transactions on Wireless Communications*, vol. 5, no. 10, pp. 2856–2862, Oct. 2006.

[46] M. Kobayashi, G. Caire, and D. Gesbert, "Transmit diversity versus opportunistic beamforming in data packet mobile downlink transmission," *IEEE Transactions on Communications,*, vol. 55, no. 1, pp. 151–157, Jan. 2007.

[47] M. Kobayashi and G. Caire, "Joint Beamforming and Scheduling for a Multi-Antenna Downlink with Imperfect Transmitter Channel Knowledge," *IEEE Journal on Selected Areas in Communications,*, vol. 25, no. 7, pp. 1468–1477, Sep. 2007.

[48] K. Huang and V. Lau, "Stability and delay of zero-forcing SDMA with limited feedback," *IEEE Transactions on Information Theory*, vol. 58, no. 10, pp. 6499–6514, Oct. 2012.

[49] J. Jose, A. Ashikhmin, P. Whiting, and S. Vishwanath, "Channel Estimation and Linear Precoding in Multiuser Multiple-Antenna TDD Systems," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 5, pp. 2102–2116, Jun. 2011.

[50] S. Lakshminarayana, B. Li, M. Assaad, A. Eryilmaz, and M. Debbah, "A fast-CSMA based distributed scheduling algorithm under SINR model," in *IEEE nternational Symposeum on Information Theory (ISIT)*, Princeton, NJ, USA, 2012, pp. 1598–1602.

[51] M. Karaca, E. Ekici, and Ö. Erçetin, "Throughput-optimal distributed algorithm for wlans over fading channels," *ArXiv e-prints*, vol. abs/1301.2750, 2013.

[52] B. Li and A. Eryilmaz, "Distributed channel probing for efficient transmission scheduling over wireless fading channels," in *IEEE International Conference on Computer Communications (INFOCOM)*, Mar. 2012, pp. 3026 –3030.

[53] A. Eryilmaz, R. Srikant, and J. R. Perkins, "Stable scheduling policies for fading wireless channels," *IEEE/ACM Transactions on Networking*, vol. 13, no. 2, pp. 411–424, Apr. 2005.

[54] K. Kar, X. Luo, and S. Sarkar, "Throughput-optimal scheduling in multichannel access point networks under infrequent channel measurements," *IEEE Transactions on Wireless Communications*, vol. 7, no. 7, pp. 2619–2629, Jul. 2008.

[55] M. Andrews, K. Kumaran, K. Ramanan, A. Stolyar, R. Vijayakumar, and P. Whiting, "Scheduling in a queueing system with asynchronously varying service rates," *Probability in the Engineering and Informational Sciences*, vol. 18, no. 2, pp. 191–217, Apr. 2004.

[56] A. Gopalan, C. Caramanis, and S. Shakkottai, "On wireless scheduling with partial channel-state information," *IEEE Transactions on Information Theory*, vol. 58, no. 1, pp. 403 –420, Jan. 2012.

[57] D. Gesbert and M.-S. Alouini, "How much feedback is multi-user diversity really worth?" in *IEEE International Conference on Communications (ICC)*, vol. 1, Paris, France, 2004, pp. 234–238.

[58] P. Svedman, S. K. Wilson, L. J. Cimini, and B. Ottersten, "Opportunistic beamforming and scheduling for OFDMA systems," *IEEE Transactions on Communications*, vol. 55, no. 5, pp. 941 –952, May 2007.

[59] S. Donthi and N. Mehta, "Joint performance analysis of channel quality indicator feedback schemes and frequency-domain scheduling for lte," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 7, pp. 3096–3109, 2011.

[60] T. Tang and R. Heath, "Opportunistic feedback for downlink multiuser diversity," *IEEE Communications Letters*, vol. 9, no. 10, pp. 948–950, 2005.

[61] J. Chen, R. Berry, and M. Honig, "Limited feedback schemes for downlink OFDMA based on sub-channel groups," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 8, pp. 1451 –1461, Oct. 2008.

[62] M. Kaneko, K. Hayashi, P. Popovski, H. Yomo, and H. Sakai, "Uplink contention-based CSI feedback with prioritized layers for a multi-carrier system," *IEEE Transactions on Wireless Communications*, vol. 10, no. 12, pp. 4282 –4293, Dec. 2011.

[63] J. Chen and V. Lau, "Large Deviation Delay Analysis of Queue-Aware Multi-User MIMO Systems With Two-Timescale Mobile-Driven Feedback," *IEEE Transactions on Signal Processing*, vol. 61, no. 16, pp. 4067–4076, 2013.

[64] S. J. Baek and G. de Veciana, "Opportunistic feedback and scheduling to reduce packet delays in heterogeneous wireless systems," *IEEE Transactions on Vehicular Technology*, vol. 61, no. 7, pp. 3282 –3289, Sept. 2012.

[65] M. Ouyang and L. Ying, "On optimal feedback allocation in multichannel wireless downlinks," in *ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*, 2010, pp. 241–250.

[66] P. Chaporkar, A. Proutiere, H. Asnani, and A. Karandikar, "Scheduling with limited information in wireless systems," in *ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*, 2009, pp. 75–84.

[67] M. Karaca, Y. Sarikaya, O. Ercetin, T. Alpcan, and H. Boche, "Efficient wireless scheduling with limited channel feedback and performance guarantees," in *IEEE International Symposeum on Personal, Indoor and Mobile Radio Communications (PIMRC)*, Sept. 2012, pp. 106 –111.

[68] ——, "Joint Opportunistic Scheduling and Selective Channel Feedback," *IEEE Transactions on Wireless Communications*, vol. 12, no. 6, pp. 3024–3034, Jun. 2013.

[69] F. Zhang and V. K. N. Lau, "Asymptotically Optimal Beamforming for Video Streaming in Multi-Antenna Interference Networks," *ArXiv e-prints*, Sep. 2013.

[70] H. Shirani-Mehr, G. Caire, and M. Neely, "MIMO Downlink Scheduling with Non-Perfect Channel State Knowledge," *IEEE Trans.actions on Communications*, vol. 58, no. 7, pp. 2055–2066, Jul. 2010.

[71] B. Ji, G. Gupta, X. Lin, and N. Shroff, "Performance of low-complexity greedy scheduling policies in multi-channel wireless networks: Optimal throughput and near-optimal delay," in *IEEE International Conference on Computer Communications (INFOCOM)*, Turin, Italy, 2013, pp. 2589–2597.

[72] L. Ying, R. Srikant, A. Eryilmaz, and G. Dullerud, "A large deviations analysis of scheduling in wireless networks," *IEEE Transactions on Information Theory*, vol. 52, no. 11, pp. 5088–5098, Nov. 2006.

[73] A. Stolyar, "Large deviations of queues sharing a randomly time-varying server," *Queueing Systems*, vol. 59, no. 1, pp. 1–35, 2008.

[74] A. Gopalan, C. Caramanis, and S. Shakkottai, "Low-delay wireless scheduling with partial channel-state information," in *IEEE International Conference on Computer Communications (INFOCOM)*, Mar. 2012, pp. 1071 – 1079.

[75] D. Wu and R. Negi, "Effective capacity: a wireless link model for support of quality of service," *IEEE Transactions on Wireless Communications*, vol. 2, no. 4, pp. 630–643, Jul. 2003.

[76] I.-H. Hou and P. Kumar, "Scheduling heterogeneous real-time traffic over fading wireless channels," in *IEEE International Conference on Computer Communications (INFOCOM)*, San Diego, CA, USA, 2010, pp. 1–9.

[77] ——, "Real-time communication over unreliable wireless links: a theory and its applications," *IEEE Wireless Communications*, vol. 19, no. 1, pp. 48–59, Feb. 2012.

[78] R. Li and A. Eryilmaz, "Scheduling for end-to-end deadline-constrained traffic with reliability requirements in multihop networks," *IEEE/ACM Transactions on Networking*, vol. 20, no. 5, pp. 1649–1662, Oct. 2012.

[79] T. Bonald, S. Borst, N. Hegde, M. Jonckheere, and A. Proutiere, "Flow-level performance and capacity of wireless networks with user mobility," *Queueing Systems*, vol. 63, no. 1-4, pp. 131–164, 2009.

[80] P. van de Ven, S. Borst, and S. Shneer, "Instability of maxweight scheduling algorithms," in *IEEE International Conference on Computer Communications (INFOCOM)*, Toronto, Canada, April 2009, pp. 1701–1709.

[81] J. Tadrous, A. Eryilmaz, and H. El Gamal, "Proactive resource allocation: Harnessing the diversity and multicast gains," *IEEE Transactions on Information Theory*, vol. 59, no. 8, pp. 4833–4854, Aug 2013.

[82] K. Shanmugam, N. Golrezaei, A. Dimakis, A. Molisch, and G. Caire, "Femtocaching: Wireless content delivery through distributed caching helpers," *IEEE Transactions on Information Theory*, vol. 59, no. 12, pp. 8402–8413, Dec. 2013.

[83] I. Karatzas and S. E. Shreve, *Brownian motion and stochastic calculus, 2nd edition*. New York: Springer-Verlag, 1991, vol. 113.

[84] H. Chen and D. D. Yao, *Fundamentals of queueing networks : performance, asymptotics, and optimization*, ser. Applications of mathematics. New York: Springer, 2001.

[85] J. G. Dai, "On positive harris recurrence of multiclass queueing networks: A unified approach via fluid limit models," *Annals of Applied Probability*, vol. 5, no. 1, pp. 49–77, 02 1995.

[86] S. P. Meyn, *Control techniques for complex networks*. Cambridge University Press, 2008.

[87] B. Øksendal, *Stochastic differential equations, 5th edition*. Heidelberg and New York: Springer-Verlag, 1999.

[88] L. Tassiulas and A. Ephremides, "Dynamic server allocation to parallel queues with randomly varying connectivity," *IEEE Transactions on Information Theory*, vol. 39, no. 2, pp. 466–478, 1993.

[89] B. Sadiq, S. J. Baek, and G. de Veciana, "Delay-optimal opportunistic scheduling and approximations: The log rule," *IEEE/ACM Transactions on Networking*, vol. 19, no. 2, pp. 405–418, 2011.

[90] S. Meyn and R. L. Tweedie, *Markov Chains and Stochastic Stability*, 2nd ed. New York, NY, USA: Cambridge University Press, 2009.

[91] M. J. Neely, E. Modiano, and C. E. Rohrs, "Power allocation and routing in multibeam satellites with time-varying channels," *IEEE/ACM Transactions on Networking*, vol. 11, no. 1, pp. 138–152, Feb. 2003.

[92] H. S. Wang and N. Moayeri, "Finite-state Markov channel-a useful model for radio communication channels," *IEEE Transactions on Vehicular Technology*, vol. 44, no. 1, pp. 163 –171, Feb. 1995.

[93] D. Tse and P. Viswanath, *Fundamentals of Wirless Communications.* New York, NY, USA: Cambridge University Press, 2005.

[94] A. R. Ward and P. W. Glynn, "Properties of the Reflected Ornstein – Uhlenbeck Process," *Queueing Systems*, vol. 44, pp. 109–123, 2003.

[95] S. Lakshminaryana, J. Hoydis, M. Debbah, and M. Assaad, "Asymptotic analysis of distributed multi-cell beamforming," in *IEEE Symposeum on Personal Indoor and Mobile Radio Communications (PIMRC)*, Sept. 2010.

[96] C. W. Gardiner, *Handbook of stochastic methods : for physics, chemistry and the natural sciences*, ser. Springer series in synergetics, 13. Springer, 2002.

[97] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 3, pp. 528–541, Mar. 2006.

[98] D. Shah and J. Shin, "Randomized scheduling algorithm for queueing networks." *Annals of Applied Probability*, vol. 22, no. 1, pp. 128–171, 2012.

[99] S. S. Nam, H.-C. Yang, M.-S. Alouini, and D. I. Kim, "Exact capture probability analysis of GSC receivers over i.n.d. rayleigh fading channels," in *IEEE International Symposium on Information Theory (ISIT), 2013*, July 2013, pp. 56–60.

[100] ——, "Technical report: An MGF-based unified framework to determine the joint statistics of partial sums of ordered i.n.d. random variables," *ArXiV e-prints*, vol. abs/1307.8199, 2013.

[101] N. Jindal, J. Andrews, and S. Weber, "Multi-antenna communication in ad hoc networks: Achieving mimo gains with simo transmission," *IEEE Transactions on Communications*, vol. 59, no. 2, pp. 529–540, Feb. 2011.

[102] R. Couillet and M. Debbah, *Random Matrix Methods for Wireless Communications.* New York, NY, USA: Cambridge University Press, 2011.

[103] T. Marzetta and B. Hochwald, "Capacity of a mobile multiple-antenna communication link in Rayleigh flat fading," *IEEE Transactions on Information Theory*, vol. 45, no. 1, pp. 139–157, Jan. 1999.

[104] B. Hassibi and T. Marzetta, "Multiple-antennas and isotropically random unitary inputs: the received signal density in closed form," *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1473–1484, Jun. 2002.

[105] V. Hassel, D. Gesbert, M.-S. Alouini, and G. Oien, "A threshold-based channel state feedback algorithm for modern cellular systems," *IEEE Transactions on Wireless Communications*, vol. 6, no. 7, pp. 2422 –2426, july 2007.

[106] A. Reddy, S. Banerjee, A. Gopalan, S. Shakkottai, and L. Ying, "On distributed scheduling with heterogeneously delayed network-state information," *Queueing Systems*, vol. 72, no. 3-4, pp. 193–218, 2012.